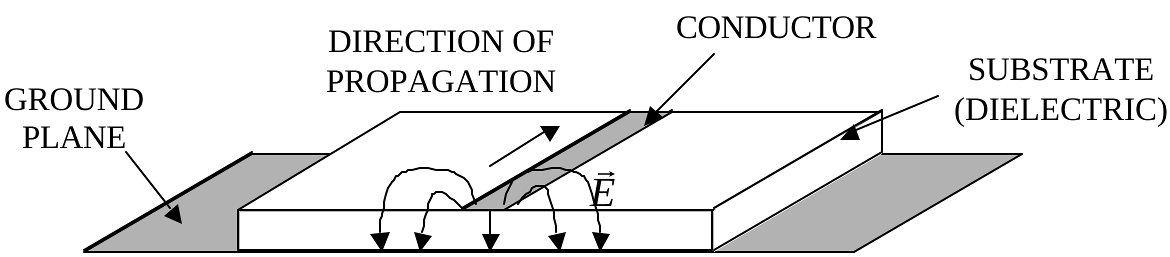
Antennas & Propagation

LECTURE NOTES
VOLUME I

REVIEW OF PLANE WAVES, TRANSMISSION LINES AND WAVEGUIDES

by Professor David Jenn

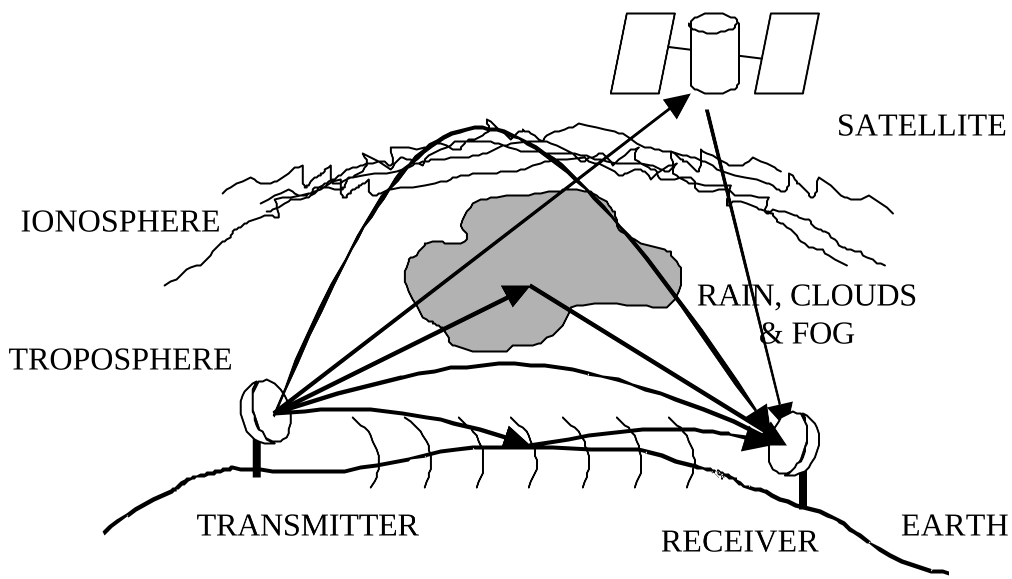


(ver 1.3)

Antennas & Propagation (1)

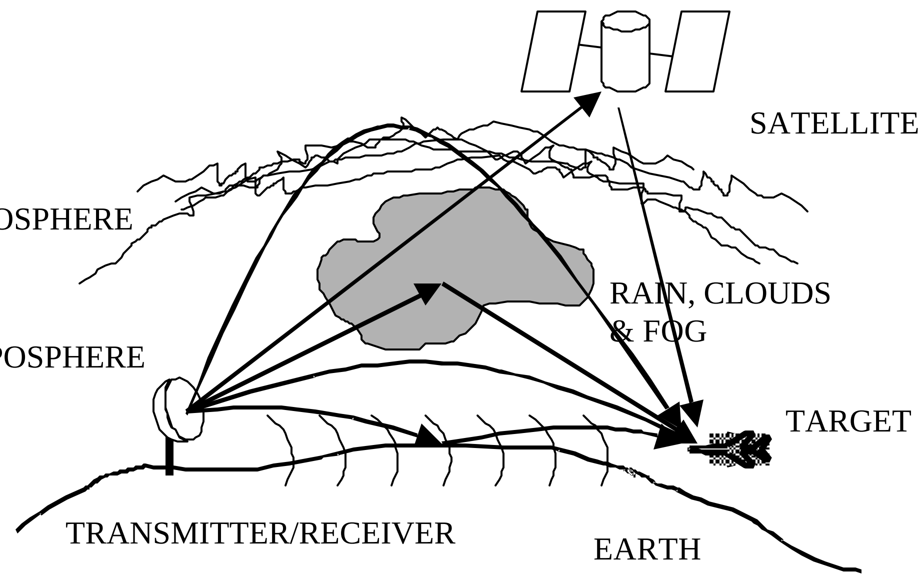
Antennas and propagation play a key role in the performance of communication, radar and electronic warfare systems. Antennas serve as transitions from guided wave structures to free space.

Example of a free-space communication channel:



Antennas & Propagation (2)

Example of a monostatic radar:



Topics of interest: general EM wave propagation
transmission lines
antennas
propagation
system issues involving antennas & propagation

Electromagnetic Fields and Waves (1)

Electrical properties of a medium are specified by its <u>constitutive parameters</u>:

- permeability, $m = m_o m_r$ (for free space, $m \equiv m_o = 4p \times 10^{-7}$ H/m)
- permittivity, $e = e_o e_r$ (for free space, $e \equiv e_o = 8.85 \times 10^{-12}$ F/m)
- conductivity, s (for a metal, $s \sim 10^{7}$ S/m)

Electric and magnetic field intensities are $\vec{E}(x,y,z,t)$ V/m and $\vec{H}(x,y,z,t)$ A/m

- they are vector functions space and time, e.g., in cartesian coordinates

$$\vec{E}(x,y,z,t) = \hat{x}E_x(x,y,z,t) + \hat{y}E_y(x,y,z,t) + \hat{z}E_z(x,y,z,t)$$

- similar expressions for other coordinates systems
- fields arise from currents $\vec{J}$ and charges r_v on the source ($\vec{J}$ is the volume current density in A/m^2 and r_v is volume charge density in C/m^3)

Electromagnetic fields are completely described by Maxwell's equations:

$$(1)\ \nabla \times \vec{E} = -m \frac{¶\vec{H}}{¶t} \qquad (3)\ \nabla \cdot \vec{H} = 0$$

$$(2)\ \nabla \times \vec{H} = \vec{J} + e \frac{¶\vec{E}}{¶t} \qquad (4)\ \nabla \cdot \vec{E} = r_v / e$$

Electromagnetic Fields and Waves (2)

Most sources of electromagnetic fields have a sinusoidal variation in time (<u>time-harmonic sources</u>). All of the field quantities associated with the sources will have the same sinusoidal time variation. Therefore, we suppress the time dependence for convenience, and work with a time independent quantity called a <u>phasor</u>. The two are related by

$$\vec{E}(z,t) = \Re\left\{\vec{E}(z)e^{jwt}\right\}$$

- $\vec{E}(z)$ is the phasor representation; $\vec{E}(z,t)$ is the instantaneous quantity
- $\Re\{\cdot\}$ is the real operator (i.e., "take the real part of")
- $j = \sqrt{-1}$

Since the time dependence varies as e^{jwt}, the time derivatives in Maxwell's equations can be replaced by $\P / \P t \equiv jw$ in the time-harmonic case:

$$\begin{array}{ll} (1)\ \nabla \times \vec{E} = -jwm\vec{H} & (3)\ \nabla \cdot \vec{H} = 0 \\ (2)\ \nabla \times \vec{H} = \vec{J} + jwe\vec{E} & (4)\ \nabla \cdot \vec{E} = r_v / e \end{array}$$

Any fields or waves that exist in a particular region of space must satisfy Maxwell's equations and the appropriate boundary conditions.

Derivation of the Wave Equation (1)

The wave equation in a source free region of space ($\vec{J} = 0,\ r_v = 0$) is derived by taking the curl of Maxwell's first equation:

$$\nabla\times\left(\nabla\times\vec{E}\right)=\nabla\times\left(-m\frac{\P\vec{H}}{\P\ t}\right)=-m\frac{\P}{\P\ t}\left(\nabla\times\vec{H}\right)=-m\frac{\P}{\P\ t}\left(e\frac{\P\vec{E}}{\P\ t}\right)=-me\frac{\P^{2}\vec{E}}{\P\ t^{2}}$$

where it is assumed that the medium is time invariant (m and e not time dependent). Now use the vector identity

$$\nabla\times\nabla\times\vec{E}=\nabla\underbrace{\left(\nabla\bullet\vec{E}\right)}_{=r_v=0}-\nabla^{2}\vec{E}=-\nabla^{2}\vec{E}$$

to obtain

$$-\nabla^{2}\vec{E}=-me\frac{\P^{2}\vec{E}}{\P\ t^{2}}\quad\Rightarrow\quad\nabla^{2}\vec{E}-me\frac{\P^{2}\vec{E}}{\P\ t^{2}}=0$$

In the frequency domain, using phasors, and noting that $\P/\P t\equiv jw$ yields

$$\nabla^{2}\vec{E}+w^{2}m_{c}e_{c}\vec{E}=\nabla^{2}\vec{E}+k_{c}^{2}\vec{E}=\nabla^{2}\vec{E}-g^{2}\vec{E}=0$$

Derivation of the Wave Equation (2)

The subscript "*c*" denotes the possibility of a complex quantity: $e_c = e' - je''$ and $m_c = m' - jm''$. The imaginary terms are nonzero if the medium is lossy. Also, we have defined

$$g \equiv a + jb = jk_c = jw\sqrt{m_c e_c}$$

where a = attenuation constant (Np/m) and $b = 2p / l$ = phase constant (rad/m). In free space, which is a lossless medium, the subscripts "o" are often used

$$e_c = e_o, m_c = m_o \quad \Rightarrow \quad a = 0,\ b = w\sqrt{e_o m_o}$$

Frequently *k* is used in place of b when the medium is lossless and unbounded. There is a similar wave equation that can be derived for the magnetic field intensity

$$\nabla^2 \vec{H} - g^2 \vec{H} = 0$$

The simplest solutions to the wave equations are plane waves. An example of a plane wave propagating in the *z* direction is:

$$\vec{E}(z) = \hat{x} E_o e^{-gz}$$

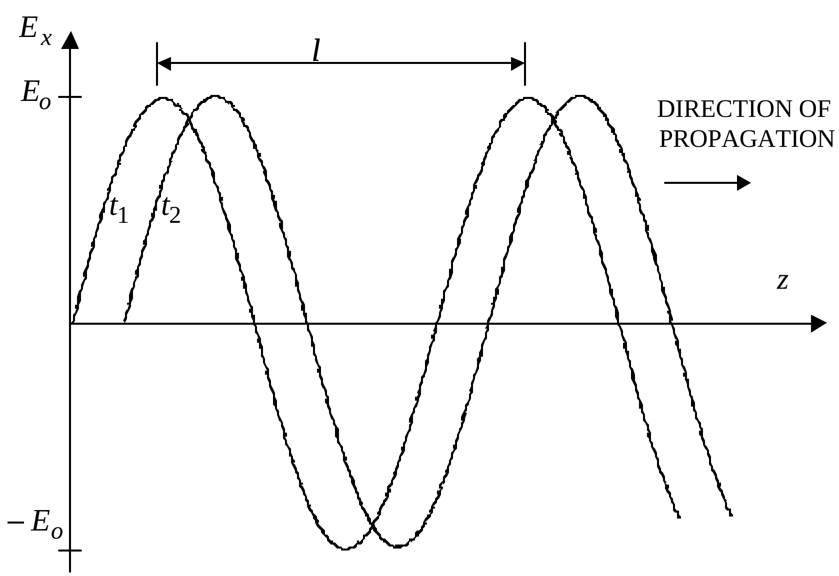
In general, E_o is a complex constant that depends on the strength of the source and its distance from the observer at *z*.

Derivation of the Wave Equation (3)

The instantaneous value of the electric field is

$$\vec{E}(z,t) = \Re\left\{\vec{E}(z)e^{jwt}\right\} = \hat{x}\,E_o e^{-az}\cos(wt - bz)$$

Time snapshots of the field are shown below



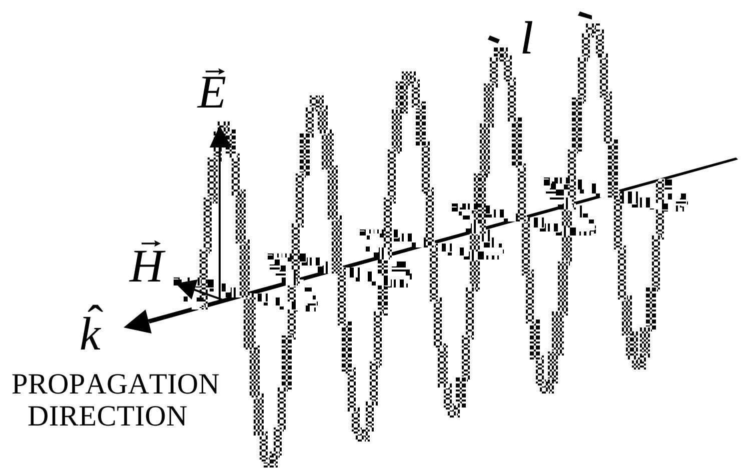
- wave propagates in the $+z$ direction
- l = wavelength
- $w = 2p\,f$ (rad/sec)
- $f = \dfrac{u_p}{l} =$ frequency (Hz)
- phase velocity is $u_p = \dfrac{w}{b} = \dfrac{1}{\sqrt{me}}$ (in free space $u_p = c = 2.998\times 10^8$ m/s)
- x polarized (direction of the electric field vector is $\hat{x}$)
- maximum amplitude of the wave is E_o

Derivation of the Wave Equation (4)

The magnetic field vector is obtained from Maxwell's first equation $\nabla \times \vec{E} = -jwm\vec{H}$

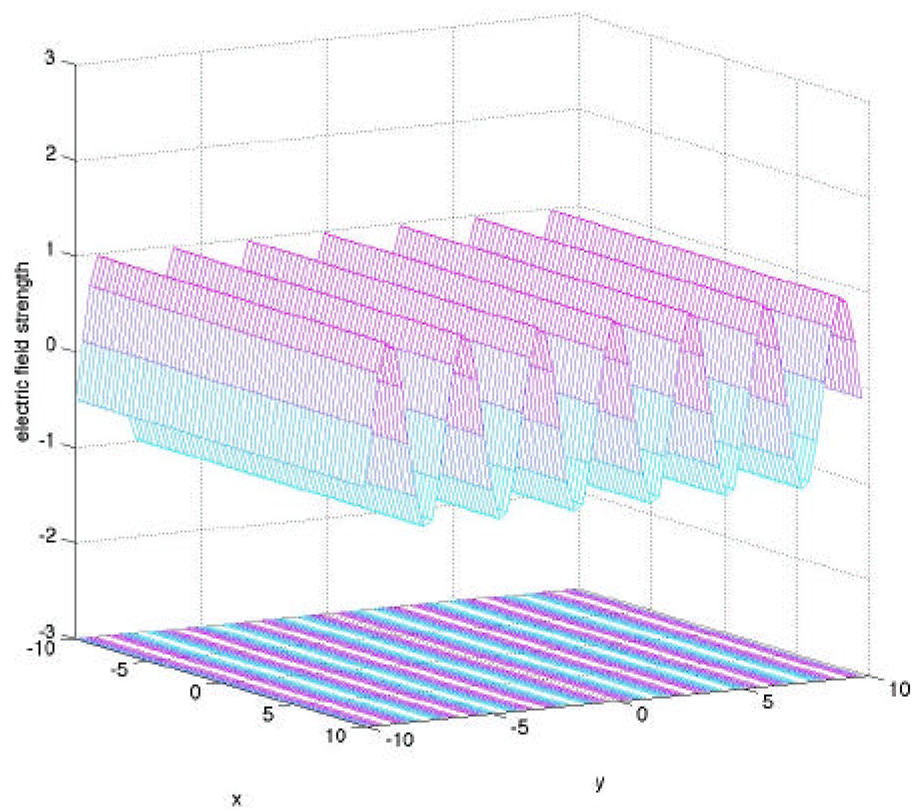
$$\vec{H} = \frac{\nabla \times \vec{E}}{-jwm} = \frac{\nabla \times \left(\hat{x} E_o e^{-jbz}\right)}{-jwm} = \frac{j}{we}\frac{\partial}{\partial z}\left(E_o e^{-jbz}\right)\hat{y} = \hat{y} \underbrace{\frac{b}{we} E_o}_{\equiv H_o} e^{-jbz}$$

The <u>intrinsic impedance</u> of the medium is $h \equiv \frac{wm}{b} = \sqrt{\frac{m}{e}} = \frac{E_o}{H_o}$. Plane waves are <u>transverse electromagnetic</u> (TEM) waves, and obey the simple relationship $\vec{H} = \frac{\hat{k} \times \vec{E}}{h}$ where $\hat{k}$ is a unit vector in the direction of propagation ($\hat{z}$ in this case). The vectors $(\hat{k}, \vec{E}, \vec{H})$ are mutually orthogonal and form a right-handed system.



Plane Wave Amplitude

Snapshot of a plane wave propagating in the $+y$ direction $\vec{E}(y,t) = \hat{z}E_o \cos(wt - by)$ at time $t = 0$



Poynting's Theorem

Poynting's theorem is a statement of conservation of energy. For a volume of space, V, bounded by a closed surface, S, and filled with a medium $(s\,, m, e\,)$

$$\underbrace{\oint_S (\vec{E}\times\vec{H})\bullet d\vec{s}}_{\text{POWER FLOWING THROUGH } S} = \underbrace{-\frac{\partial}{\partial t}\int_V \left(\tfrac{1}{2}eE^2 + \tfrac{1}{2}mH^2\right)dv}_{\text{POWER STORED IN THE FIELDS INSIDE OF } S} - \underbrace{\int_V sE^2\,dv}_{\text{POWER LOSS IN } S}$$

The quantity $\vec{W} = \vec{E}\times\vec{H}$ $(\mathrm{W/m}^2)$ is known as the Poynting vector. The instantaneous value of the Poynting vector is

$$\vec{W}(x,y,z,t) = \vec{E}(x,y,z,t)\times\vec{H}(x,y,z,t) = \Re\left\{\vec{E}(x,y,z)e^{jwt}\right\}\times\Re\left\{\vec{H}(x,y,z)e^{jwt}\right\}$$

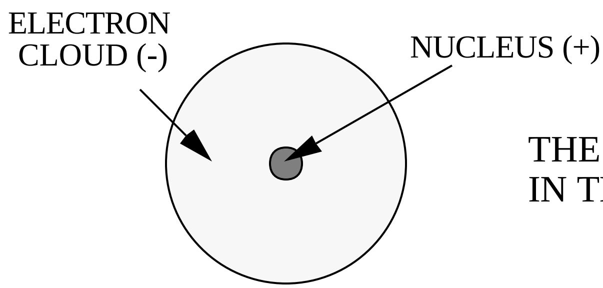
and the time-averaged Poynting vector is

$$\vec{W}_{\mathrm{av}} = \frac{1}{T}\int_0^T \vec{W}(x,y,z,t)\,dt = \frac{1}{2}\Re\left\{\vec{E}(x,y,z)\times\vec{H}(x,y,z)^*\right\}$$

The time-averaged value can be found directly from the phasor fields quantities.

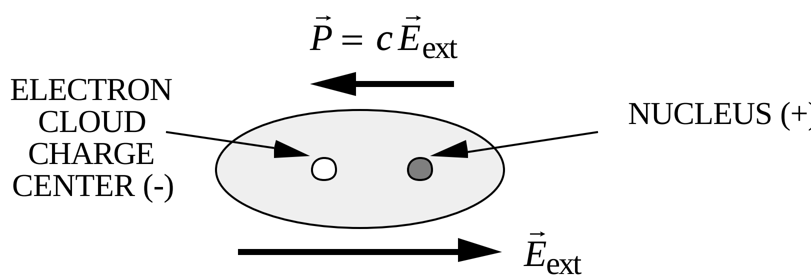
Debye Model (1)

The Debye model has been used to predict the interaction of EM waves with materials since about 1910. Molecules are represented by positive and negative charge centers.



THE CHARGE CENTERS ARE COINCIDENT IN THE ABSENCE OF AN EXTERNAL FIELD

The response of a molecule to an external electric field is expressed in terms of a polarization, $\vec{P}(t)$



This is the simplest form of a dipole: two equal and opposite charges that are slightly displaced. The separation that arises due to the external field is referred to as the <u>electronic polarization</u> and the quantity c is the <u>susceptibility</u>.

Debye Model (2)

It takes a finite amount of time for the molecules to respond to an applied external field. The response is of the form

$$P(t) = \underbrace{P_o}_{c(0)E_{\text{ext}}} \; e^{-t/t}$$

where t is the <u>relaxation constant</u> (about 10^{-15} second).

Assumptions are that all dipoles are identical, independent, and all relaxation times are the same. In fact, dipoles are spatially and temporally coupled, relaxation times vary, and other types of polarization exist. The Debye model is never seen in real materials, but it can be approached for single particle non-interacting systems such as gases.

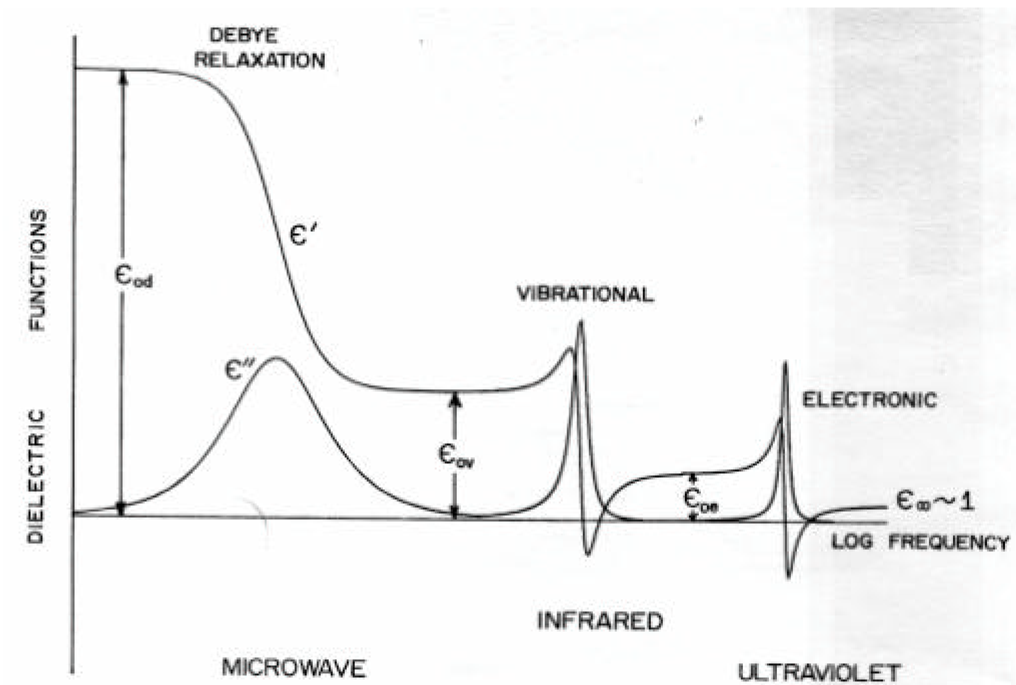
Other types of polarization:

<u>Ionic</u>: mutual displacement of the charge centers (10^{-13} second)
<u>Orientational</u>: rotation of the molecules (10^{-11} second)

Media have a far more complex EM relaxation behavior than previously realized. A new theory (Dissado-Hill) takes all of these factors into account.

Permittivity of a Dielectric With Loss

- Example of a material with resonances in the millimeter wave frequency region
- Complex dielectric constant: $e_c = e' - je''$
- Below millimeter wave frequencies, $e' \geq 1$ and approximately constant and $e'' \approx 0$



(From Bohren and Huffman, *Absorption and Scattering of Light by Small Particles*)

- Phase velocity (e' is the real part)

$$u_p = \frac{1}{\sqrt{me'}} = \frac{w}{b}$$

High frequencies travel faster than low frequencies

Propagation in Lossy Media (1)

As waves propagate through a lossy medium, energy is extracted from the wave and absorbed by the medium. There are three general sources of loss:

1. ohmic loss, which is due to the collision of free charges in a conductor, and is accounted for by a finite conductivity, $s < \infty$ ($s = \infty$ is a perfect electric conductor, PEC)
2. dielectric loss, due to polarization of molecules caused by an external electric field, and it is accounted for in the imaginary part of e_c
3. magnetic loss, due to magnetization of the molecules caused by an external magnetic field, and it is accounted for in the imaginary part of m_c

Most materials are non-magnetic ($m = m_o$) and therefore magnetic losses can be neglected. For all other materials, either ohmic loss or dielectric loss dominates. For an imperfect conductor, an equivalent complex dielectric constant can be derived by introducing the conduction current into Maxwell's second equation

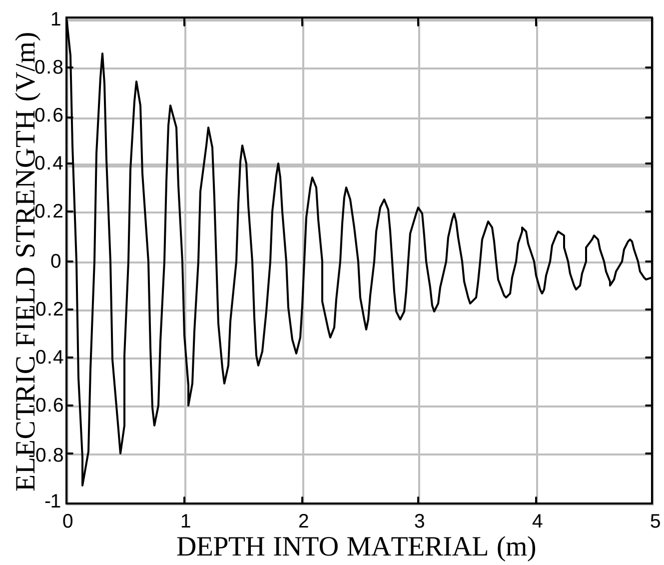
$$\begin{aligned} \nabla \times \vec{H} &= \vec{J} + s\vec{E} + jwe\vec{E} \\ &= \vec{J} + jw\underbrace{\left(e + \frac{s}{jw}\right)}_{e_c}\vec{E} \end{aligned}$$

Propagation in Lossy Media (2)

The attenuation constant determines the rate of decay of the wave. General formulas for the attenuation and phase constants of a conductor are:

$$\alpha = \omega \left\{ \frac{\mu\varepsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon} \right)^2} - 1 \right] \right\}^{1/2} \qquad \beta = \omega \left\{ \frac{\mu\varepsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon} \right)^2} + 1 \right] \right\}^{1/2}$$

For lossless media $\sigma = 0 \Rightarrow \alpha = 0$. Traditionally, for lossless cases, k is used rather than β. For <u>good conductors</u> ($\sigma / \omega\varepsilon >> 1$), $\alpha \approx \sqrt{\pi \mu f \sigma}$, and the wave decays rapidly with distance into the material. A sample plot of field vs. distance is shown.

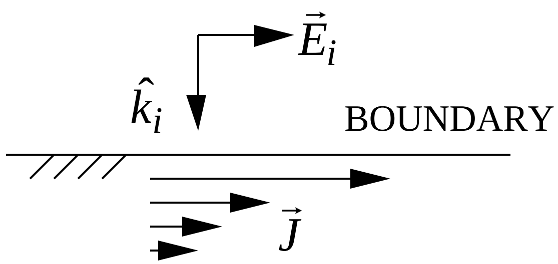


(To apply the formulas to a dielectic with losses, substitute $\varepsilon \rightarrow \varepsilon'$ and $\sigma / \omega \rightarrow \varepsilon''$.)

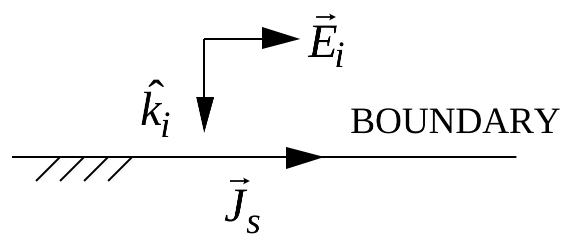
Surface Current and Resistivity (1)

For good conductors the current is concentrated near the surface. The current can be approximated by an infinitely thin current sheet, with <u>surface current density</u>, $\vec{J}_s$ A/m and <u>surface charge density</u>, r_s C/m^2

Current in a good conductor



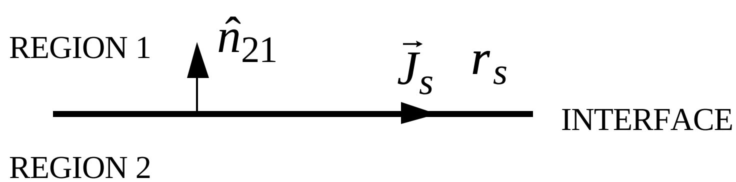
Surface current approximation



At an interface between two media the boundary conditions must be satisfied:

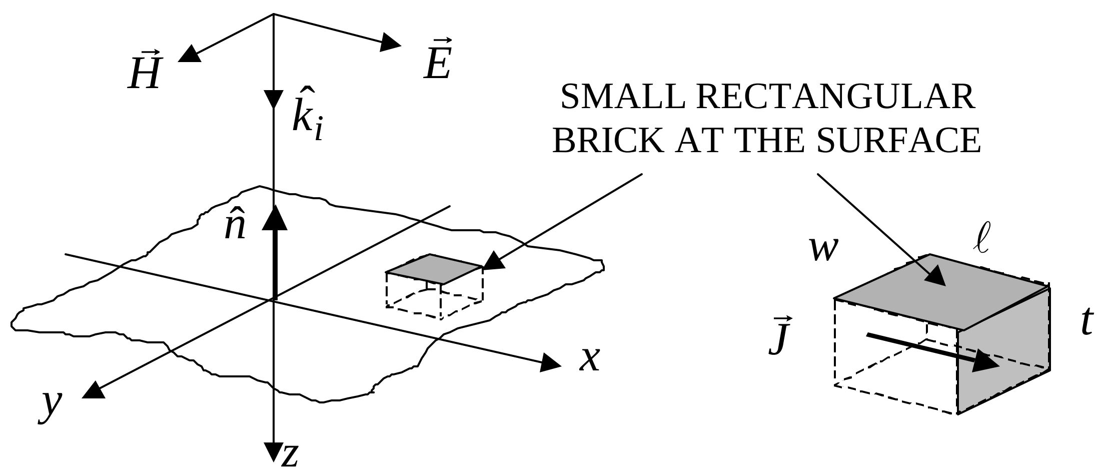
$$(1)\ \hat{n}_{21} \times (\vec{E}_1 - \vec{E}_2) = 0 \qquad (3)\ \hat{n}_{21} \cdot (\vec{E}_1 - \vec{E}_2) = r_s / e$$

$$(2)\ \hat{n}_{21} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s \qquad (4)\ \hat{n}_{21} \cdot (\vec{H}_1 - \vec{H}_2) = 0$$



Surface Current and Resistivity (2)

The field in a good conductor is significant only within the first skin depth from the surface. The skin depth is the distance into the material at which the amplitude has decayed by a factor of 1/*e*.



The resistance of the block is $R = \dfrac{\ell}{sA} = \dfrac{\ell}{stw}$, where A is the cross sectional area transverse to the direction of current flow. If we choose a square of surface area, $\ell = w$, and the thickness a skin depth d_s, then the result is the surface resistivity, which is defined as $R_s = \dfrac{1}{s d_s}$. It has units of "ohms per square" ($\Omega/\square$)

Surface Current and Resistivity (3)

For a plane wave normally incident on a metal surface, the time-averaged power density in the material is

$$\vec{W}_{\mathrm{av}} = \frac{1}{2}\Re\left\{\vec{E}\times\vec{H}^*\right\} = \frac{1}{2}\Re\left\{\frac{E_o^2}{h^*} e^{-2az}\right\}\hat{z} = \frac{E_o^2}{2|h|^2} e^{-2az}\Re\{h\} = \hat{z}\,\frac{\hat{z} R E_o^2}{2|h|^2} e^{-2az}$$

It is assumed that E_o is real for convenience. For a good conductor the intrinsic impedance is approximately

$$h \equiv R + jX \approx \frac{1+j}{s d_s}$$

(Note that the real part is equal to the surface resistivity previously defined.) We can replace the original infinitely thick medium with an infinitesimally thin sheet that satisfies the same boundary condition:

$$\hat{n}\times\vec{H} = \vec{J}_s \quad \rightarrow \quad \hat{n}\times\hat{n}\times\vec{H} = \hat{n}\times\vec{J}_s \quad \rightarrow \quad \hat{n}\underbrace{\left(\hat{n}\bullet\vec{H}\right)}_{=0} - \vec{H}\left(\hat{n}\bullet\hat{n}\right) = \hat{n}\times\vec{J}_s$$

where $\vec{H} = \dfrac{\hat{k}_i\times\vec{E}}{h_s} = \dfrac{-\hat{n}\times\vec{E}}{h_s}$ and h_s is the <u>surface impedance</u> of the thin sheet.

Surface Current and Resistivity (4)

The boundary condition can be written as

$$h_s \hat{n} \times \vec{J}_s = \hat{n} \times \vec{E} \quad \rightarrow \quad h_s \vec{J}_s = \vec{E}_{\tan}$$

and the power dissipated by the current flowing on the boundary is

$$P_{\text{loss}} = -\frac{1}{2} \Re\left\{ \vec{E} \times \vec{H}^* \right\} \bullet \hat{n} = -\frac{1}{2} \Re\left\{ \underbrace{\hat{n} \times \vec{E}}_{h_s \hat{n} \times \vec{J}_s} \bullet \vec{H}^* \right\}$$

$$= \frac{1}{2} \Re\left\{ h_s \vec{J}_s \bullet \underbrace{\hat{n} \times \vec{H}^*}_{\vec{J}_s^*} \right\} = \frac{\left|\vec{J}_s\right|^2}{2} \Re\{h_s\} = \frac{\left|\vec{J}_s\right|^2}{2} R_s$$

The surface impedance concept gives a convenient means of computing the ohmic loss of conductors. We can avoid integrating the volume current inside of the conductor (a volume integral), and need only integrate the surface current (a surface integral). This is only an approximation, but it is very accurate for good conductors. These calculations are necessary in order to determine transmission line loss.

Circular Polarization (1)

A circularly polarized plane wave can be obtained by superimposing two equal amplitude linearly polarized plane waves that are in <u>space and time quadrature</u> (quadrature implies 90 degrees):

1. space quadrature, $\vec{E}_1 \perp \vec{E}_2$ (for example, E_x vs. E_y)
2. phase quadrature, a $e^{\pm j\pi/2}$ factor between the two fields

Example: Two linearly polarized plane waves propagating in the *z* direction

$$\vec{E}_1 = \hat{x} E_{x_o} e^{-j\beta z} \quad \text{and} \quad \vec{E}_2 = \hat{y} E_{y_o} e^{-j\beta z} e^{\pm j\pi/2}$$

Equal amplitudes, $E_{x_o} = E_{y_o} \equiv E_o$

$$\vec{E}(z) = \vec{E}_1(z) + \vec{E}_2(z) = \hat{x} E_o e^{-j\beta z} + \hat{y} E_o e^{-j\beta z} \underbrace{e^{\pm j\pi/2}}_{\pm j} = E_o e^{-j\beta z} \left(\hat{x} \pm j\,\hat{y} \right)$$

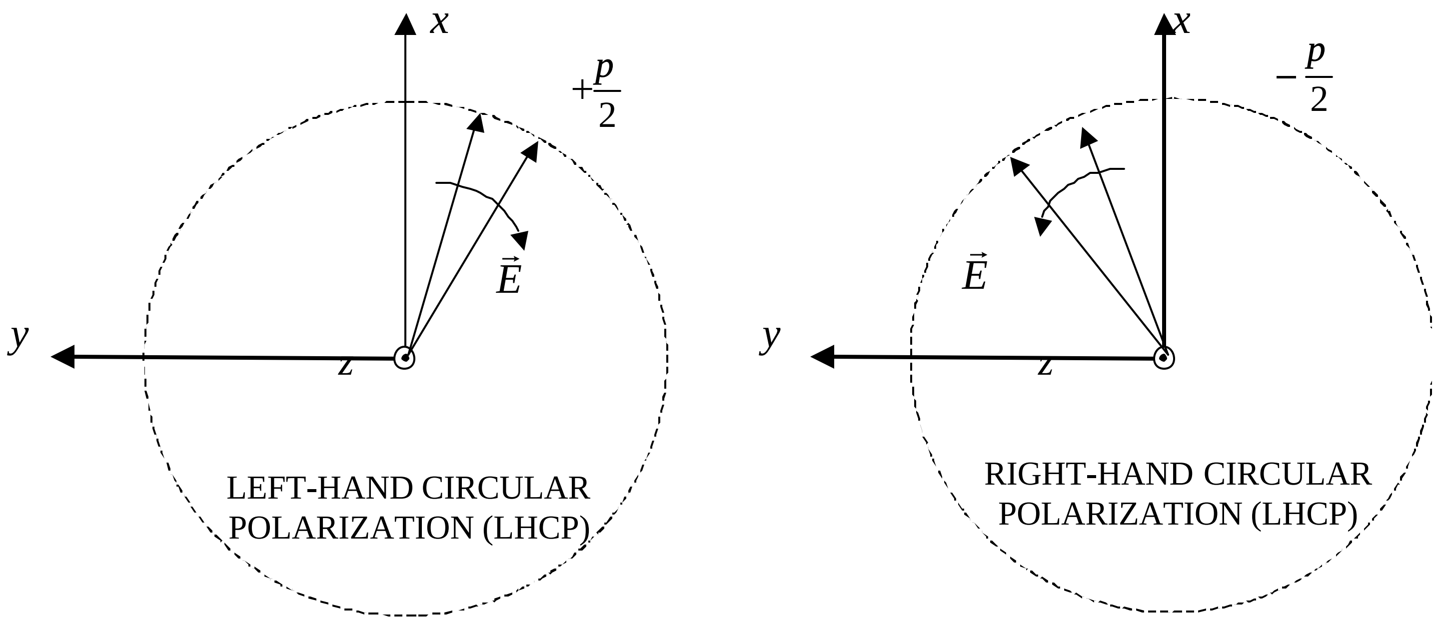
The instantaneous value at $z = 0$ is

$$\vec{E}(z,t) = \Re\left\{ \vec{E}(z) e^{j\omega t} \right\} = \hat{x} E_o \cos(\omega t) \mp \hat{y} E_o \sin(\omega t)$$

The vector rotates about the *z* axis. The tip of the electric field vector traces out a circle of radius E_o. The direction of rotation depends on the sign of *j*.

Circular Polarization (2)

The designation of RHCP is determined by the right-hand rule: the thumb of the right hand is pointed in the direction of propagation, and the fingertips give the direction of rotation of the electric field vector. Similarly, LHCP satisfies the left-hand rule.



The above signs hold for e^{-jbz}. If $E_{x_o} \neq E_{y_o}$ then the tip of the electric field vector traces an ellipse. The resulting polarization is referred to as elliptical polarization.

Example

We want to find the reflected field when a RHCP plane wave is normally incident on a flat perfectly conducting surface,

$$\vec{E}_i(z) = \hat{x}E_o e^{-jbz} - j\hat{y}E_o e^{-jbz}$$

Assume that the reflected field is of the form

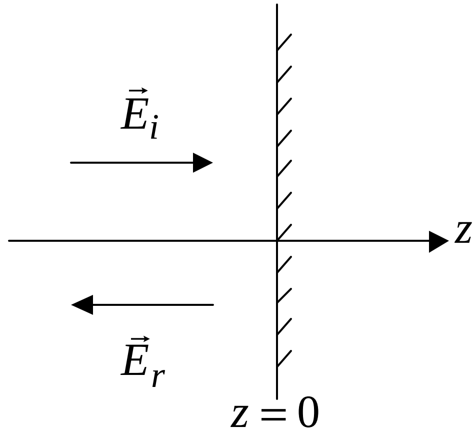
$$\vec{E}_r(z) = \hat{x}E_{rx} e^{+jbz} + \hat{y}E_{ry} e^{+jbz}$$

The total tangential field at the boundary ($z = 0$) must be zero

$$\vec{E}_i(z) + \vec{E}_r(z) = \hat{x}\left(E_o + E_{rx}\right) + \hat{y}\left(E_{ry} - jE_o\right) \equiv 0$$

Equate x and y components to obtain

$$E_{rx} = -E_o$$
$$E_{ry} = jE_o$$



The total field is

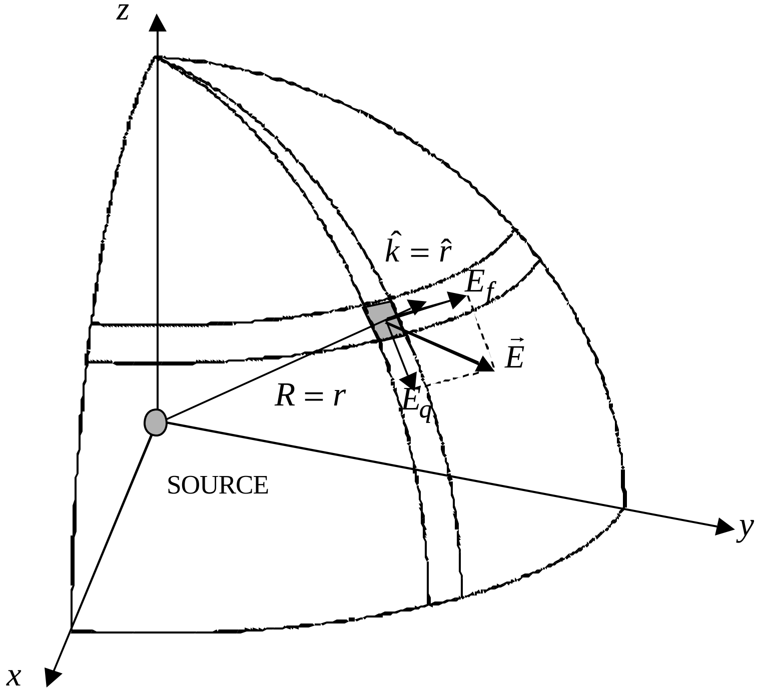
$$\vec{E}_r(z) = -\hat{x}E_o e^{+jbz} + j\hat{y}E_o e^{+jbz}$$

which is a LHCP wave.

Spherical Waves (1)

An ideal point source for electromagnetic waves has no volume. It radiates a spherical wave (i.e., the equiphase planes are spherical surfaces). An arbitrarily polarized spherical wave can written as

$$\vec{E}(R) = \frac{e^{-jbR}}{R}\left(E_{qo}\hat{q} + E_{fo}\hat{f}\right)$$



- R= distance from the source (Note that if the source is at the origin of the spherical coordinate system then R=r. Thus we will move the source to the origin and use r in the next few charts.)
- h =impedance of the medium, assumed to be real
- $b = \dfrac{2p}{l}$
- E_{qo}, E_{fo} are complex constants

Spherical Waves (2)

Spherical waves are TEM, so the magnetic field intensity is

$$\vec{H}(r) = \frac{\hat{k}\times\vec{E}(r)}{h} = \frac{\hat{r}\times\left(E_{qo}\hat{q} + E_{fo}\hat{f}\right)}{hr}e^{-jbr} = \frac{e^{-jbr}}{hr}\left(-E_{fo}\hat{q} + E_{qo}\hat{f}\right)$$

and the time-averaged Poynting vector (assuming E_{qo}, E_{fo} are real)

$$\vec{W}_{\mathrm{av}} = \frac{1}{2}\vec{E}(r)\times\vec{H}^*(r) = \frac{1}{2r^2h}\left(E_{qo}\hat{q} + E_{fo}\hat{f}\right)\times\left(-E_{fo}\hat{q} + E_{qo}\hat{f}\right)^*$$

$$= \frac{1}{2r^2h}\left((E_{qo})^2 + (E_{fo})^2\right)\hat{r}$$

The power flowing through a spherical surface of radius r is

$$P_{\mathrm{av}} = \iint_S \vec{W}_{\mathrm{av}} \bullet ds = \frac{1}{2h}\left((E_{qo})^2 + (E_{fo})^2\right)\int_0^{2p}\int_0^{p}\frac{1}{r^2}\hat{r}\bullet\hat{r}\,r^2\sin q\,dq\,df$$

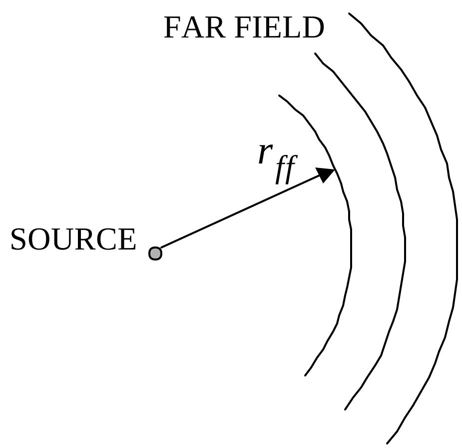
$$= \frac{2p}{2h}\left((E_{qo})^2 + (E_{fo})^2\right)\underbrace{\int_0^{p}\sin q\,dq}_{=2} = \frac{2p}{h}\left((E_{qo})^2 + (E_{fo})^2\right)$$

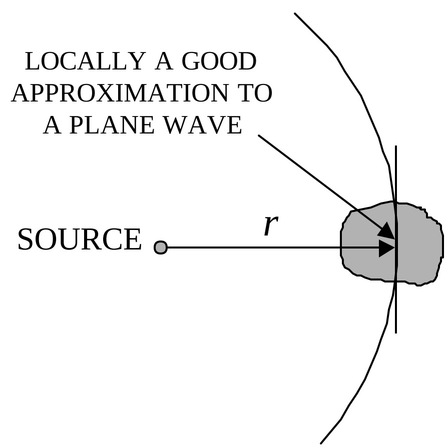
Spherical Waves (3)

Note that the power spreads as $\frac{1}{r^2}$ (the "inverse square law"). We will see that a far field region can be defined for any antenna. It is the region beyond a minimum distance, r_{ff}, where the wave becomes spherical with the following properties:

1. the wave propagates radially outward
2. it is TEM (there are only q and f field components)
3. the field components vary as $\frac{1}{r}$

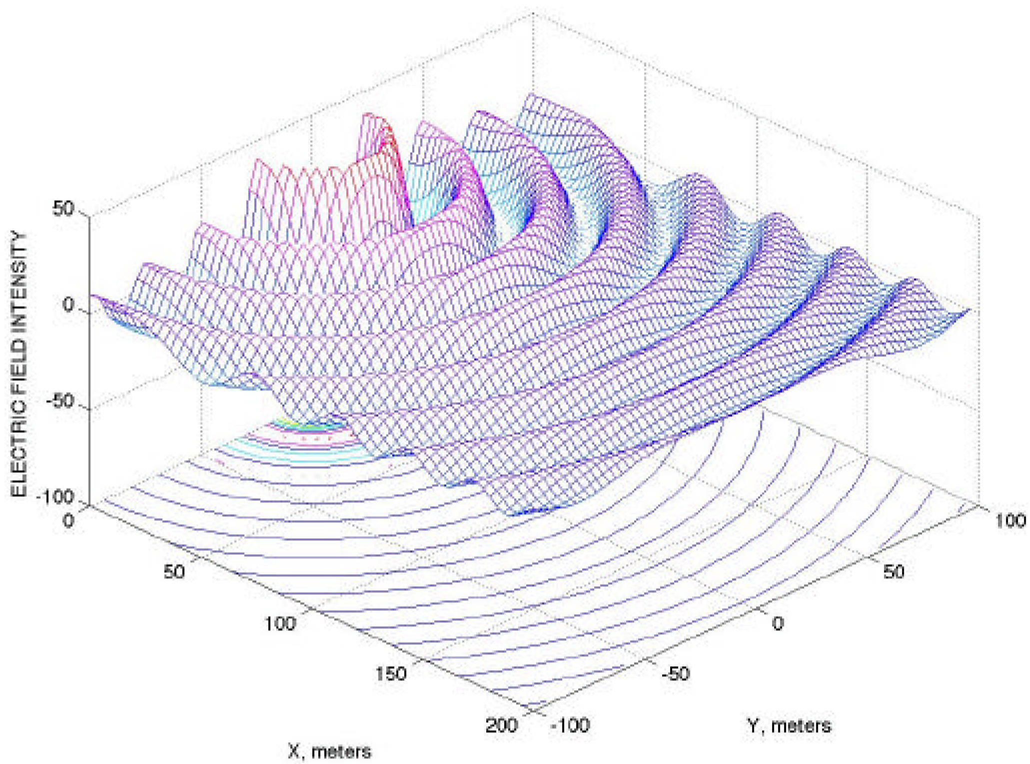
At a large distance from the source of a spherical wave, the phase front becomes locally plane.





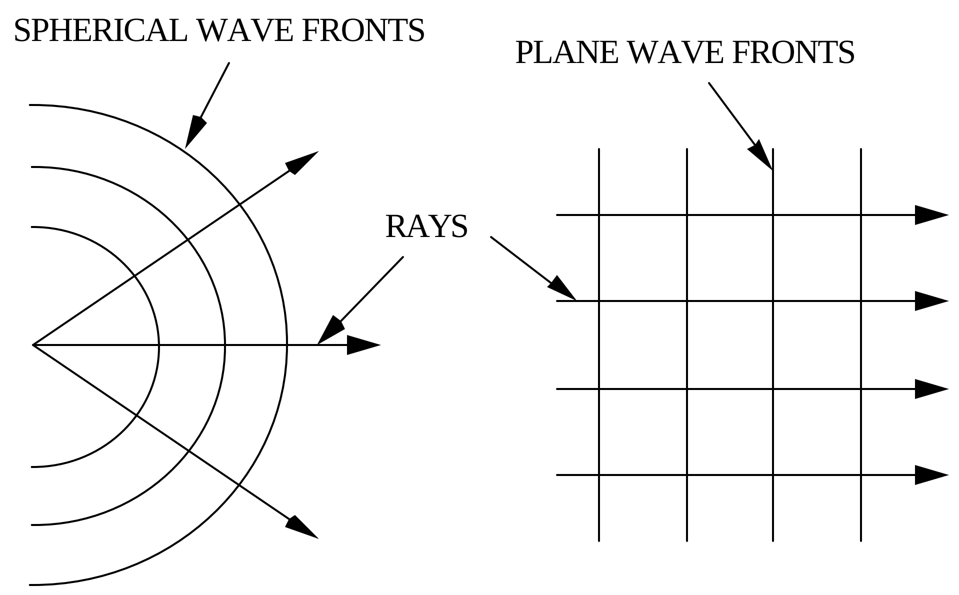
Spherical Wave Amplitude

Snapshot of a spherical wave propagating outward from the origin. The amplitude of the wave $\vec{E}(r,t) = \hat{q}\,\frac{E_o}{r}\cos(wt - br)$ in the x-y plane is plotted at time $t = 0$



Ray Representation for Waves

- Rays are often used to represent a propagating wave. They are arrows in the direction of propagation ($\hat{k}$) and are everywhere perpendicular to the equiphase planes (wavefronts)
- The behavior of rays upon reflection or refraction is given by a set of rules which form the basis of <u>geometrical optics</u> (the classical theory of ray tracing)
- We will see that if an observer gets far enough from a finite source of radiation, then the wavefronts become spherical
- At even larger distances the wavefronts become approximately planar on a local scale



Wave Reflection (1)

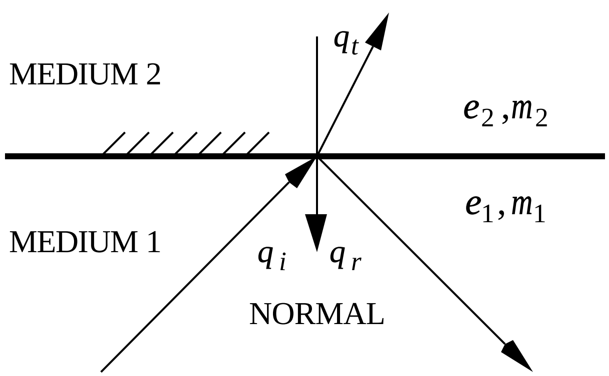
For the purposes of applying boundary conditions, the electric field vector is decomposed into parallel and perpendicular components $\vec{E} = \vec{E}_{\perp} + \vec{E}_{\parallel}$

$\vec{E}_{\perp}$ is perpendicular to the plane of incidence

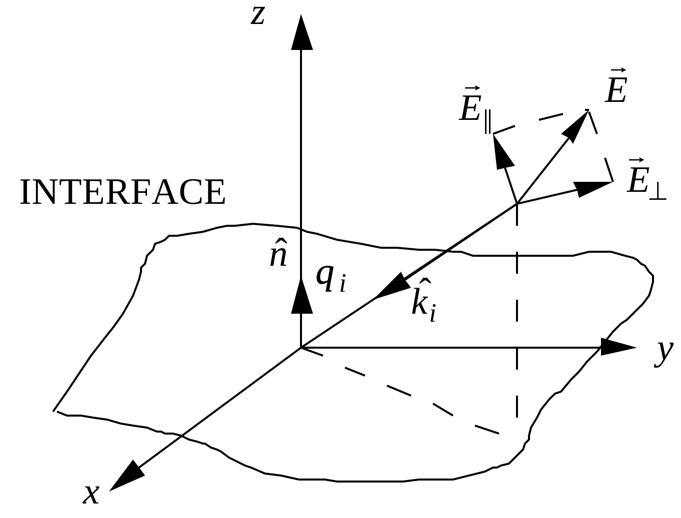
$\vec{E}_{\parallel}$ lies in the plane of incidence

The plane of incidence is defined by the vectors $\hat{k}_i$ and $\hat{n}$

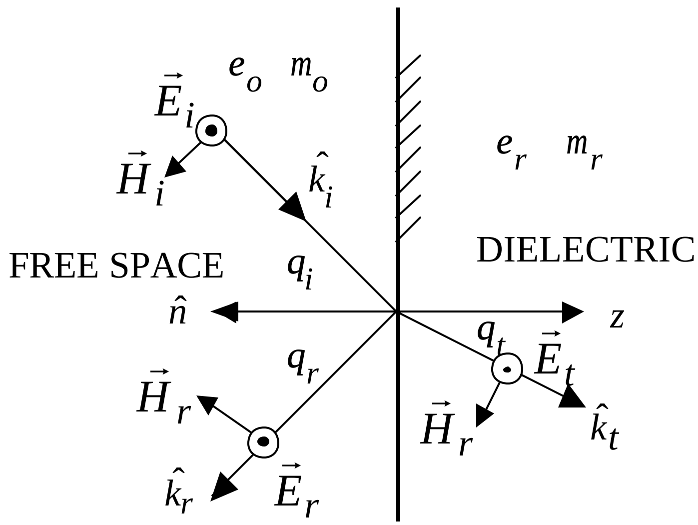
PLANE WAVE INCIDENT ON AN INTERFACE BETWEEN TWO DIELECTRICS



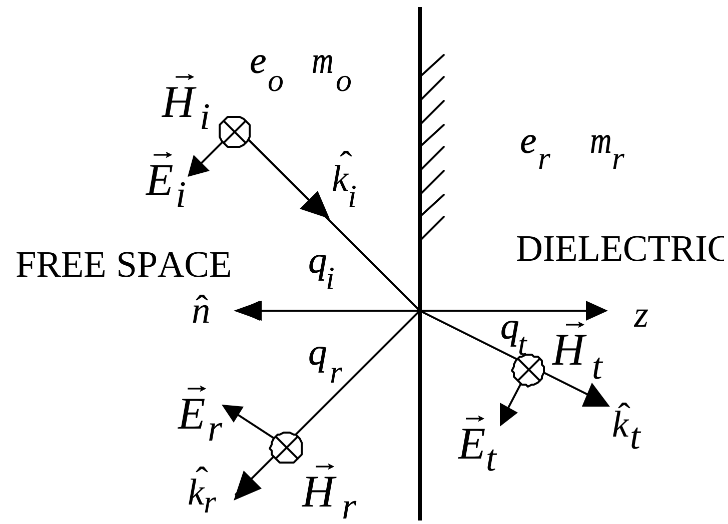
DECOMPOSITON OF AN ELECTRIC FIELD VECTOR INTO PARALLEL AND PERPENDICULAR COMPONENTS



Wave Reflection (2)



PERPENDICULAR POLARIZATION



PARALLEL POLARIZATION

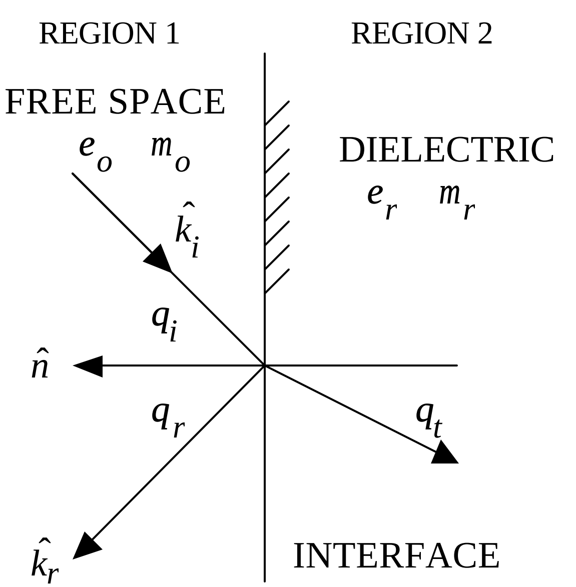
The incident fields $(\vec{E}_i, \vec{H}_i)$ are known in each case. We can write expressions for the reflected and transmitted fields $(\vec{E}_r, \vec{H}_r)$ and $(\vec{E}_t, \vec{H}_t)$, and then apply the boundary conditions at $z = 0$:

$$\left(\vec{E}_i + \vec{E}_r\right)\Big|_{\tan} = \left(\vec{E}_t\right)\Big|_{\tan} \text{ and } \left(\vec{H}_i + \vec{H}_r\right)\Big|_{\tan} = \left(\vec{H}_t\right)\Big|_{\tan}$$

There is enough information to solve for the coefficients of the reflected and transmitted waves.

Wave Reflection (3)

Summary of results:



$$\sin q_i = \sin q_r = \sqrt{e_r m_r} \sin q_t$$

$$h_o = \sqrt{\frac{m_o}{e_o}} \text{ and } h = \sqrt{\frac{m_r m_o}{e_r e_o}} = h_o \sqrt{\frac{m_r}{e_r}}$$

Reflection and transmission coefficients:

Perpendicular polarization:

$$\Gamma_\perp = \frac{h \cos q_i - h_0 \cos q_t}{h \cos q_i + h_0 \cos q_t}$$

$$t_\perp = \frac{2h \cos q_i}{h \cos q_i + h_0 \cos q_t}$$

$$E_{r\perp} = \Gamma_\perp E_{i\perp} \text{ and } E_{t\perp} = t_\perp E_{i\perp}$$

Parallel polarization:

$$\Gamma_\parallel = \frac{h \cos q_t - h_0 \cos q_i}{h \cos q_t + h_0 \cos q_i}$$

$$t_\parallel = \frac{2h \cos q_i}{h \cos q_t + h_0 \cos q_i}$$

$$E_{r\parallel} = \Gamma_\parallel E_{i\parallel} \text{ and } E_{t\parallel} = t_\parallel E_{i\parallel}$$

Wave Reflection (4)

Example: A boundary between air ($\eta_o = 377\,\Omega$) and glass ($\epsilon_r = 4, \eta = 188.5\,\Omega$).
See the following charts for plots of reflection coefficients vs. incidence angle.

Two special cases:

1. Brewster's angle is the incidence angle at which the reflection coefficient is zero. For parallel polarization this requires $\eta \cos\theta_t - \eta_o \cos\theta_i = 0$, or

$$\cos\theta_i = \frac{\eta}{\eta_o}\cos\theta_t = \frac{\eta}{\eta_o}\sqrt{1-\sin^2\theta_t} = \frac{\eta}{\eta_o}\sqrt{1-\frac{1}{\epsilon_r}\sin^2\theta_i} \;\Rightarrow\; \theta_B = \tan^{-1}\sqrt{\epsilon_r} = 63.4^\circ$$

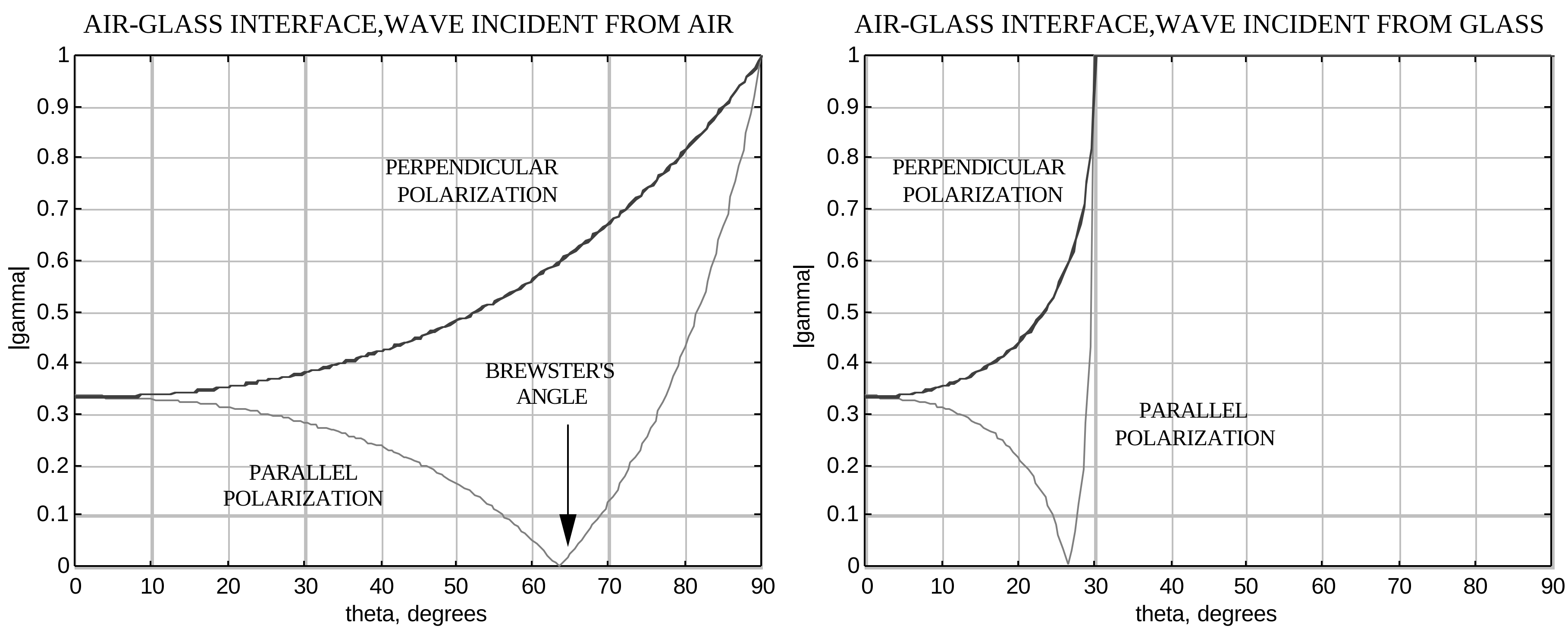
2. Total internal reflection ($\theta_t = \pi/2$) occurs at the critical angle of incidence, when the wave is impinging on the boundary from the more dense medium

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu\epsilon}{\mu_o\epsilon_o}} \;\Rightarrow\; \frac{\sin(\pi/2)}{\sin\theta_c} = \sqrt{\frac{\mu\epsilon}{\mu_o\epsilon_o}} = \sqrt{\epsilon_r} \;\Rightarrow\; \sin\theta_c = \frac{1}{2} \;\Rightarrow\; \theta_c = 30^\circ$$

This is the basis of fiber optic transmission lines.

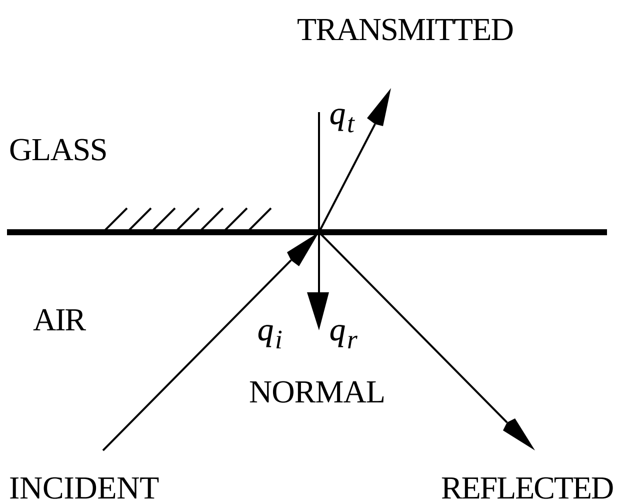
Wave Reflection (5)

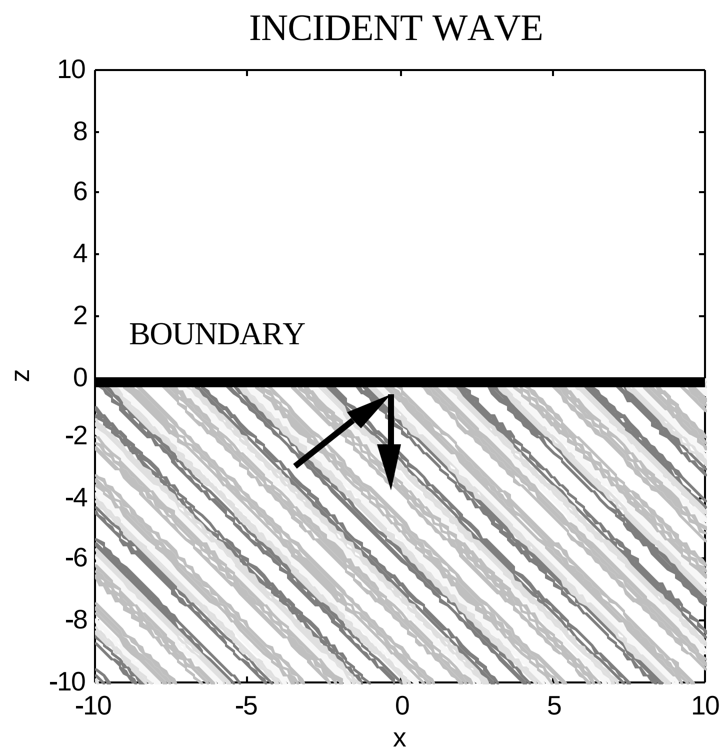
Boundary between air $(e_r = 1)$ and glass $(e_r = 4)$



Wave Reflection (6)

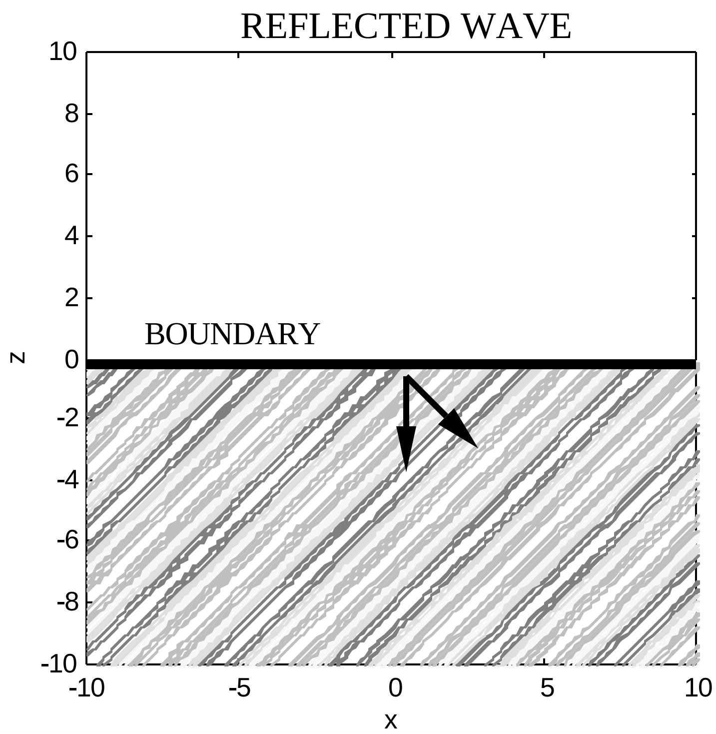
Example of a plane wave incident on a boundary between air and glass ($e_r = 4, q_i = 45^\circ$)

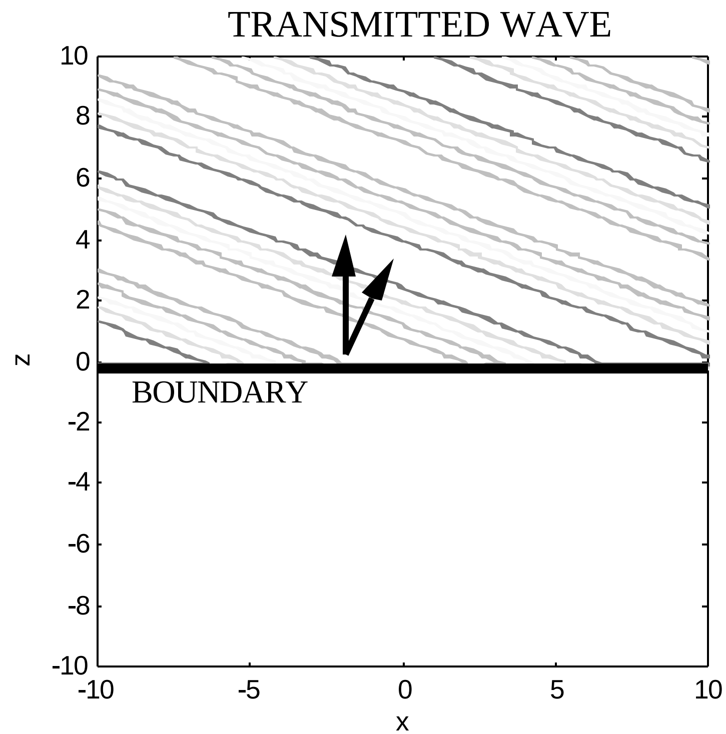




Wave Reflection (7)

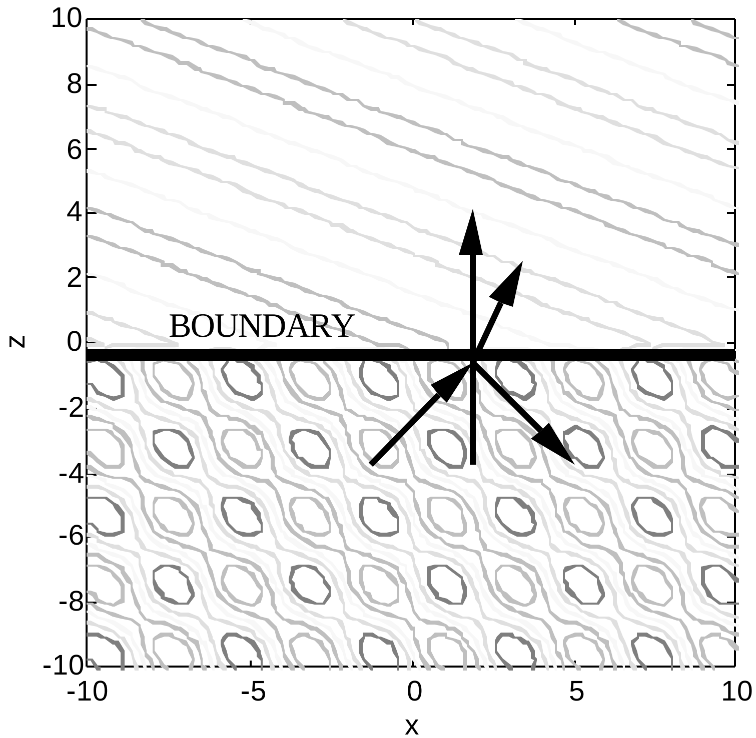
Example of a plane wave reflection: reflected and transmitted waves ($e_r = 4, q_i = 45^\circ$)





Wave Reflection (8)

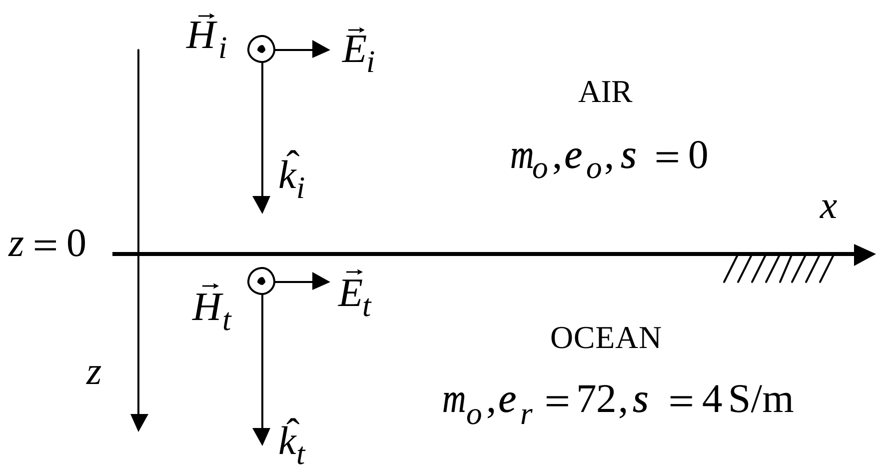
Example of a plane wave reflection: total field



- The total field in region 1 is the sum of the incident and reflected fields
- If region 2 is more dense than region 1 (i.e., $\epsilon_{r2} > \epsilon_{r1}$) the transmitted wave is refracted towards the normal
- If region 1 is more dense than region 2 (i.e., $\epsilon_{r1} > \epsilon_{r2}$) the transmitted wave is refracted away from the normal

Example (1)

An aircraft is attempting to communicate with a submerged submarine. The frequency is 0.5 MHz and the power density of the normally incident wave at the ocean surface is 12 $\mathrm{kW/m}^2$. The receiver on the submarine requires 0.1 $m\mathrm{V/m}$ to establish a reliable link. What is the maximum depth for communication?



The phasor expression for the incident plane wave is $\vec{E}_i(z) = \hat{x} E_o e^{-jb_o z}$ where

$b_o = w\sqrt{m_o e_o} = \dfrac{2p}{l_o}$, $l_o = \dfrac{c}{f} = \dfrac{3\times 10^8}{0.5\times 10^6} = 600$ m. The time-averaged power density is

given by the Poynting vector, $\left|\vec{W}_{\mathrm{av}_i}(z)\right| = \dfrac{1}{2}\left|\vec{E}_i(z)\times\vec{H}_i^*(z)\right|$

Example (2)

At the ocean surface, z=0, and from the information provided we can solve for E_o

$$\left|\vec{W}_{\text{av}_i}(z)\right| = \frac{|E_o|^2}{2h_o} \equiv 12\times 10^3 \text{ W/m}^2 \quad \Rightarrow \quad |E_o|^2 = (12\times 10^3)(2)(377) \quad \Rightarrow \quad |E_o| = 3008 \text{ V/m}$$

Below the ocean surface the electric field is given by $\vec{E}_t(z) = \hat{x}E_o t\, e^{-g\, z}$, where the transmission coefficient is determined from the Fresnel formulas

$$\Gamma = \frac{h - h_0}{h + h_0}, \text{ and } t = 1 + \Gamma = \frac{2h}{h + h_o}.$$

To evaluate this we need the impedance of seawater

$$h = \sqrt{\frac{m}{e_c}} = \sqrt{\frac{m_o}{e - j\dfrac{s}{w}}} = \sqrt{\frac{m_o}{e_o e_r\left(1 - j\dfrac{s}{e_o e_r w}\right)}}$$

Note that $\dfrac{s}{e_o e_r w} = \dfrac{4}{2p(5\times 10^5)(8.85\times 10^{-12})(72)} = 2000 \gg 1$ which is typical of a good conductor. Thus we drop the 1 in the denominator for good conductors.

Example (3)

$$h \approx \sqrt{j}\sqrt{\frac{w m_o}{s}} = \frac{1+j}{\sqrt{2}}\sqrt{\frac{w m_o}{s}} = 0.7(1+j) = 0.9899\, e^{j45^\circ} \equiv |h| e^{j\Phi_h}$$

Now the transmission coefficient is

$$t = \frac{2h}{h + h_o} = \frac{2(1+j)(0.7)}{(1+j)(0.7)+377} = 5.25\times 10^{-3} e^{j44.89^\circ} \equiv |t| e^{j\Phi_t}$$

At depth *z* the magnitude of the electric field intensity is

$$\left|\vec{E}_t(z)\right| = \sqrt{\vec{E}_t(z) \bullet \vec{E}_t(z)^*} = \left[\left(\hat{x}E_o t\ e^{-(a+jb)z}\right) \bullet \left(\hat{x}E_o t\ e^{-(a+jb)z}\right)^*\right]^{1/2} = |E_o||t|\, e^{-a\ z}$$

where the attenuation constant is

$$a = w\left\{\frac{me}{2}\left[\sqrt{1+\left(\frac{s}{we}\right)^2} - 1\right]\right\}^{1/2}$$

$$a \approx \sqrt{\frac{w m_o s}{2}} = \sqrt{p\ f m_o s} = \sqrt{p\,(5\times 10^5)(4p\times 10^{-7})(4)} = 2.81\ \text{Np/m}$$

Example (4)

Similarly, for a good conductor the phase constant is

$$b = w\left\{\frac{me}{2}\left[\sqrt{1+\left(\frac{s}{we}\right)^2}+1\right]\right\}^{1/2} \approx \sqrt{\frac{w m_o s}{2}} = a$$

At what depth is $\left|\vec{E}_t(z)\right| = 0.1\,m$ V/m?

$$\begin{aligned} 0.1\times 10^{-6} &= |E_o||t|e^{-2.81z} = (3008)(5.24\times 10^{-3})e^{-2.81z} \\ -2.81z &= -18.88 \\ z &= 6.7 \text{ m} \end{aligned}$$

A common measure of the <u>depth of penetration</u> of the wave in a conductor is the <u>skin depth</u>, d_s. It is the distance that the wave travels into the material at which its magnitude is $1/e$ of its value at the surface

$$\left|\vec{E}_t(0)\right|e^{-1} = |E_o||t|e^{-1} = |E_o||t|e^{-a\,d_s} \quad\Rightarrow\quad a d_s = 1 \quad\Rightarrow\quad d_s = 1/a = 1/\sqrt{p\, f m s}$$

(Note that for a <u>nonmagnetic</u> conductor $m = m_o$.)

Example (5)

The instantaneous (time-dependent) expression for the field is

$$\vec{E}_t(z,t) = \Re\left\{ \hat{x}\, t E_o e^{-jb\,z} e^{jw\,t} \right\}$$

Note that in general E_o can be complex and written in polar form $E_o = |E_o| e^{j\Phi_o}$. The phase depends on the altitude of the transmitter and the phase of the wave upon leaving the aircraft antenna. We can not determine Φ_o from the information provided, and furthermore, it is not important in determining whether the link is established. Thus,

$$\vec{E}_t(z,t) = \Re\left\{ \hat{x} |t| e^{j\Phi_t} \, |E_o| e^{j\Phi_o} e^{-(a+jb)z} e^{jw\,t} \right\} = \hat{x} |t| \, |E_o| e^{-a\,z} \cos\left(w\,t - bz + \Phi_o + \Phi_t\right)$$

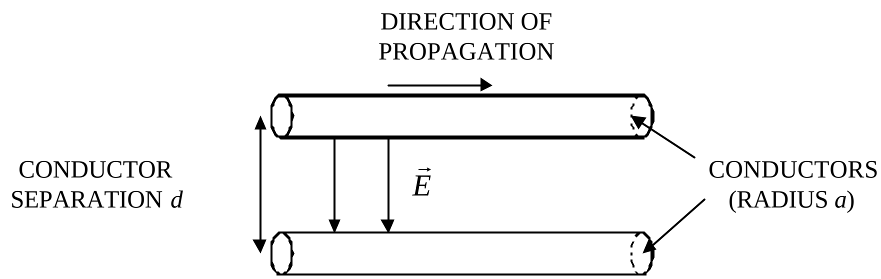
The magnetic field intensity phasor is $\vec{H}_t(z) = \dfrac{\hat{k}_t \times \vec{E}_t(z)}{h} = \dfrac{\hat{y} E_o t}{h} e^{-g\,z}$ and the instantaneous value

$$\vec{H}_t(z,t) = \Re\left\{ \frac{\hat{x} |t| e^{j\Phi_t} \, |E_o| e^{j\Phi_o}}{|h| e^{j\Phi_h}} e^{-(a+jb)z} e^{jw\,t} \right\}$$

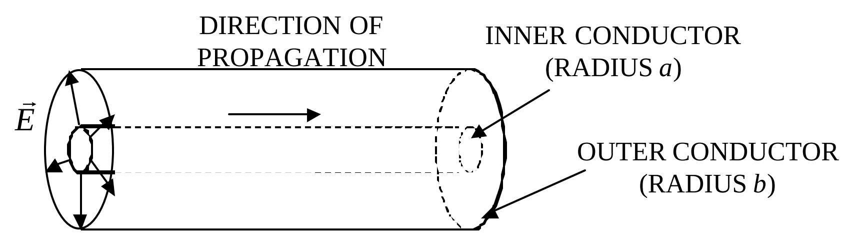
$$= \frac{\hat{x} |t| \, |E_o|}{|h|} e^{-a\,z} \cos\left(w\,t - bz + \Phi_o + \Phi_t - \Phi_h\right)$$

Common Two-Wire Transmission Lines

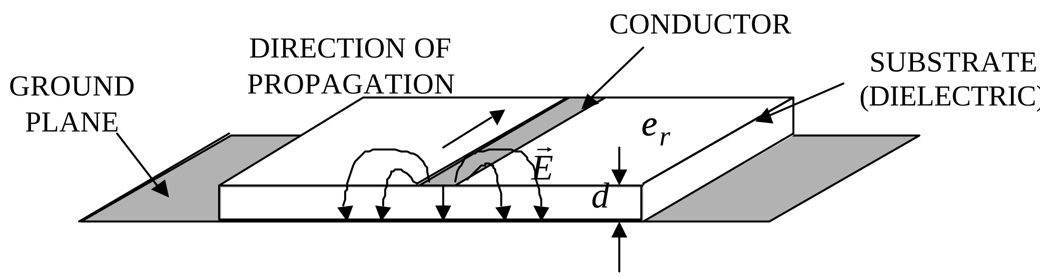
Twin lead or two-wire



Coaxial ("coax")

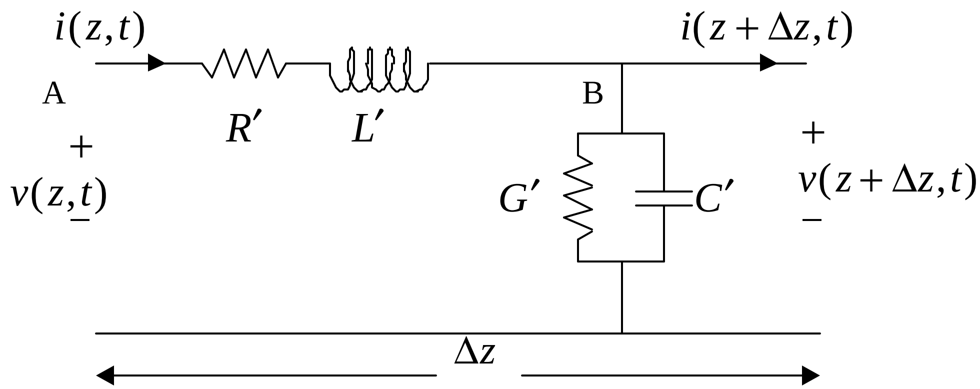


Microstrip



Transmission Line Equations (1)

A short length (Δz) of a two-wire transmission line has the equivalent circuit shown below:



R' is the total resistance of the conductors (Ω/m)
L' is the inductance due to the magnetic field around the conductors (H/m)
C' is the series capacitance due to the electric field between the conductors (V/m)
G' is the is the conductance due to loss in the material between the conductors (S/m)

Special case: lossless transmission line

1. perfect conductors, $s = \infty$ and therefore $R' = 0$
2. perfect dielectric filling the region between the conductors, $e'' = 0$ and therefore $G' = 0$

Transmission Line Equations (2)

Use Kirchhoff's voltage law at node A and take $\lim\limits_{\Delta z \to 0}$

$$-\frac{\partial v(z,t)}{\partial z} = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t}$$

Use Kirchhoff's current law at node B and take $\lim\limits_{\Delta z \to 0}$

$$-\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t}$$

For the time-harmonic case $\partial / \partial t \to jw$

$$-\frac{dV(z)}{dz} = \left(R' + jwL'\right) I(z) \qquad (1)$$

$$-\frac{dI(z)}{dz} = \left(G' + jwC'\right) V(z) \qquad (2)$$

This is a set of coupled integral equations. Take d/dz of (1) and substitute it in (2) to get a second order differential equation for $V(z)$

$$\frac{d^2 V(z)}{dz^2} - \underbrace{\left(R' + jwL'\right)\left(G' + jwC'\right)}_{\equiv g^2} V(z) = 0$$

Transmission Line Equations (3)

The propagation constant is determined from the transmission line parameters

$$g = \sqrt{(R' + jwL')(G' + jwC')} \equiv a + jb$$

The phase velocity is $u_p = w / b$.

In a similar manner a differential equation can be derived for the current. Together they are the transmission line equations (wave equations specialized to transmission lines)

$$\frac{d^2V(z)}{dz^2} - g^2 V(z) = 0 \quad \text{and} \quad \frac{d^2 I(z)}{dz^2} - g^2 I(z) = 0$$

A solution for the voltage is

$$V(z) = V_o^+ e^{-gz} + V_o^- e^{+gz}$$

The first term is a wave traveling in the $+z$ direction and the second a wave traveling in the -z direction. If this is inserted into (1) on the previous page then the result is

$$I(z) = \frac{g}{R' + jwL'}\left(V_o^+ e^{-gz} - V_o^- e^{+gz}\right)$$

Transmission Line Equations (4)

The corresponding solution of the differential equation for the current is

$$I(z) = I_o^+ e^{-gz} + I_o^- e^{+gz}$$

Comparing the coefficients of the terms in the two equations gives the <u>characteristic impedance</u>

$$Z_o \equiv \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R' + jwL'}{g} = \frac{g}{G' + jwC'} = \sqrt{\frac{R' + jwL'}{G' + jwC'}}$$

Example: Air line ($e = e_o, m = m_o, s = 0$) with perfect conductors ($s_{\text{cond}} = \infty$) operates at 700 MHz and has a characteristic impedance of 50 ohms and a phase constant of 20 rad/m. Find L', C' and the phase velocity.

Since $R' = G' = 0 \quad \Rightarrow \quad a = 0$, and $g = \sqrt{jwL'jwC'} = jw\sqrt{L'C'} \equiv jb = j20$

$Z_o = \sqrt{\dfrac{R' + jwL'}{G' + jwC'}} = \sqrt{\dfrac{L'}{C'}} = 50$. Solve the two equations to obtain $C' = 90.9$ pF/m and $L' = 227$ nH/m. The phase velocity is $u_p = w / b = 1/\sqrt{L'C'} = 2.2 \times 10^8$ m/s $= 0.733c$.

Transmission Line Equations (5)

Formulas are available for computing the transmission line parameters of various configurations. For example, a coax with inner radius *a* and outer radius *b*:

$$G' = \frac{2ps}{\ln(b/a)}, \quad C' = \frac{2pe}{\ln(b/a)}, \quad L' = \frac{m}{2p}\ln(b/a), \quad R' = \frac{R_s}{2p}(1/a + 1/b)$$

where $R_s = \sqrt{\frac{p\ f m_{\text{cond}}}{s_{\text{cond}}}}$ is the surface resistance of the conductors, m_{cond} is its permeability and s_{cond} its conductivity. Note that m, e and s are the constitutive parameters of the material filling the medium between the conductors.

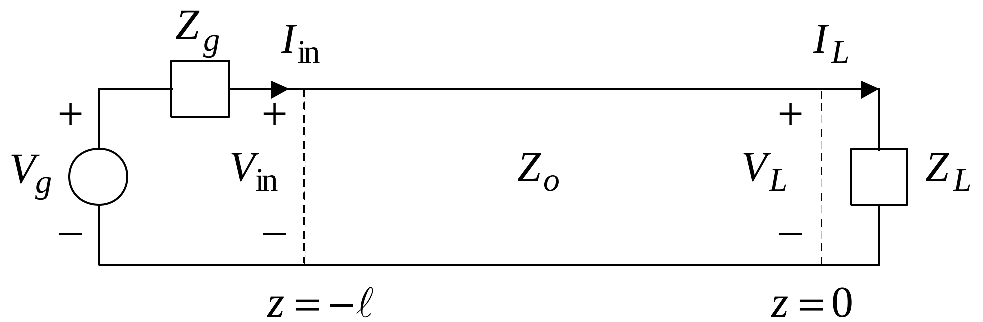
For transmission lines that support transverse electromagnetic (TEM) waves the following relationships hold:

$$L'C' = me \quad \text{and} \quad \frac{G'}{C'} = \frac{s}{e}$$

An important characteristic of TEM waves is that $\vec{E}$, $\vec{H}$ and the direction of propagation $\hat{z}$ are mutually orthogonal. That is, the electric and magnetic field vectors lie in a plane transverse to the direction of propagation.

Transmission Line Circuits (1)

A transmission line circuit is shown below. The source (generator) and receiver are connected by a length ℓ of transmission line. Assume a lossless line ($g = jb$)



The current and voltage on the line are given by

$$V(z) = V_o^+ e^{-jbz} + V_o^- e^{+jbz} \text{ and } I(z) = \frac{V_o^+}{Z_o} e^{-jbz} - \frac{V_o^-}{Z_o} e^{+jbz}$$

The boundary condition at the load ($z = 0$) can be used to derive a reflection coefficient

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_o^+ + V_o^-}{\frac{V_o^+}{Z_o} - \frac{V_o^-}{Z_o}} \quad \Rightarrow \quad \Gamma = \frac{V_o^+}{V_o^-} = \frac{Z_L - Z_o}{Z_L + Z_o} \equiv |\Gamma| e^{j\Phi_\Gamma}$$

Transmission Line Circuits (2)

Three special load conditions are:

1. If the load is <u>matched</u> to the characteristic impedance of the line then $Z_L = Z_o$ and $\Gamma = 0$
2. If the line is open circuited then $Z_L = \infty$ and $\Gamma = 1$ ($|\Gamma| = 1, \Phi_\Gamma = 0$)
3. If the line is short circuited then $Z_L = 0$ and $\Gamma = -1$ ($|\Gamma| = 1, \Phi_\Gamma = p$)

The total voltage at a point on the line is given by

$$V(z) = V_o^+ e^{-jbz} + V_o^- e^{jbz} = V_o^+ \left(e^{-jbz} + \frac{V_o^-}{V_o^+} e^{jbz} \right)$$
$$= V_o^+ \left(e^{-jbz} + \Gamma e^{jbz} \right) = V_o^+ \left(e^{-jbz} + |\Gamma| e^{j\Phi_\Gamma} e^{jbz} \right)$$
$$|V(z)| = \sqrt{V(z)V(z)^*} = \left| V_o^+ \right| \left\{ 1 + |\Gamma|^2 + 2|\Gamma| \cos(2bz + \Phi_\Gamma) \right\}^{1/2}$$

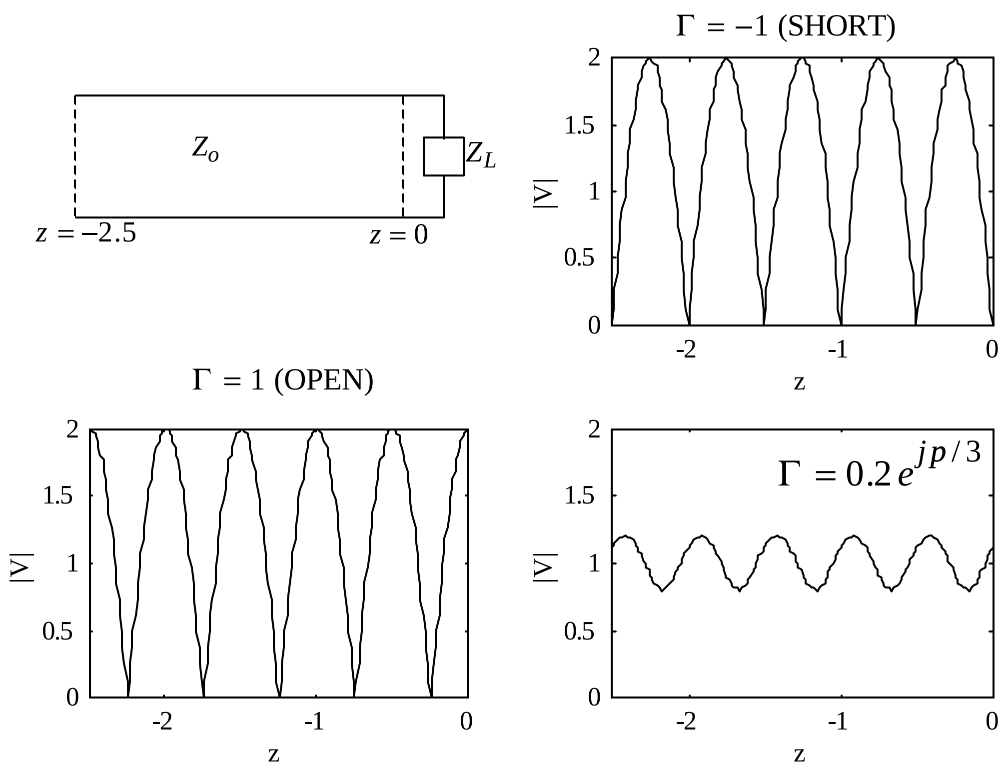
The maximum and minimum values of the voltage are

$$V_{\max} = |V|_{\max} = \left| V_o^+ \right| (1 + |\Gamma|) \quad \text{and} \quad V_{\min} = |V|_{\min} = \left| V_o^+ \right| (1 - |\Gamma|)$$

If $\Gamma \neq 0$ there is a <u>standing wave</u> component to the voltage and current.

Transmission Line Circuits (3)

Voltage plots for three load conditions ($l = 1$m):



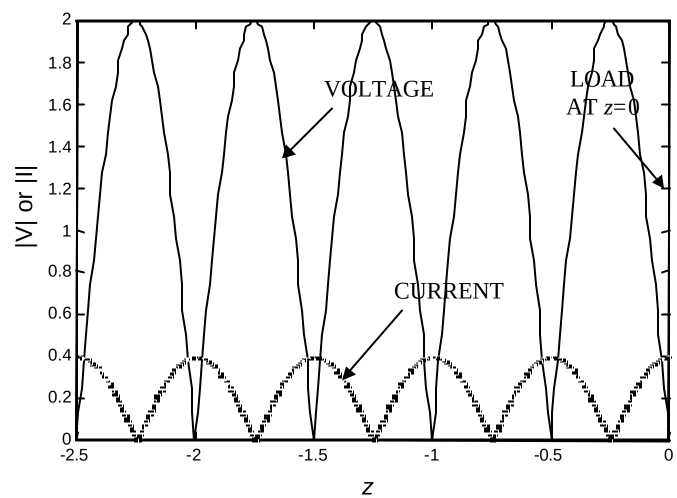
The load impedance for the last case can be computed from the reflection coefficient

$$\Gamma = \frac{Z_L - Z_o}{Z_L - Z_o} = \frac{Z_L' - 1}{Z_L' - 1} = 0.2e^{jp/3} \quad \Rightarrow \quad Z_L' = \frac{Z_L}{Z_o} = \frac{1+\Gamma}{1-\Gamma} = 1.14 + j0.41$$

Transmission Line Circuits (4)

- Voltage maxima occur when $\cos(2bz + \Phi_\Gamma) = 1 \quad \Rightarrow \quad 2bz + \Phi_\Gamma = -2np$. (Note that increasing *n* is in the –*z* direction.) Maxima are spaced $l/2$.
- Voltage minima occur when $\cos(2bz + \Phi_\Gamma) = -1 \quad \Rightarrow \quad 2bz + \Phi_\Gamma = -(2n+1)p$. Minima are spaced $l/2$.
- The <u>voltage standing wave ratio</u> (VSWR) is defined as $s = \dfrac{|V|_{\max}}{|V|_{\min}} = \dfrac{(1+|\Gamma|)}{(1-|\Gamma|)}$. Note that $1 < s < \infty$.

Plot of voltage and current for $Z_L' = 0.1$ ($l = 1$ m).



Transmission Line Impedance (1)

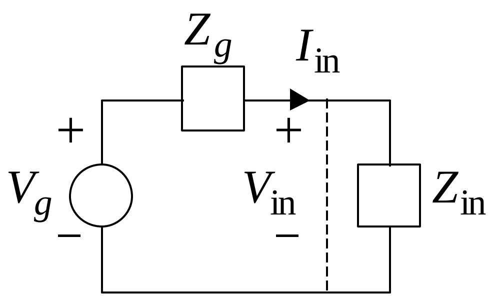
The impedance at any point on the line is the ratio of the voltage to current at that point

$$Z(z) = \frac{V(z)}{I(z)} = \frac{V_o^+\left[e^{-jbz} + \Gamma e^{+jbz}\right]}{\frac{V_o^+}{Z_o}\left[e^{-jbz} - \Gamma e^{+jbz}\right]} = Z_o \frac{1+\Gamma e^{j2bz}}{1-\Gamma e^{j2bz}}$$

At the input of the line $z = -\ell$

$$Z_{\text{in}} = Z_o \frac{1+\Gamma e^{-j2b\ell}}{1-\Gamma e^{-j2b\ell}} = Z_o\left[\frac{Z_L + jZ_o \tan(b\ell)}{Z_o + jZ_L \tan(b\ell)}\right]$$

For the purpose of computing the power delivered to the load, the load and transmission line can be replaced by an equivalent impedance Z_{in}



$$I_{\text{in}} = \frac{V_g}{Z_g + Z_{\text{in}}} \quad \text{and} \quad V_{\text{in}} = I_{\text{in}} Z_{\text{in}} = \frac{V_g Z_{\text{in}}}{Z_g + Z_{\text{in}}}$$

The power delivered to the load and line combination is $P_{\text{in}} = V_{\text{in}} I_{\text{in}}^*$

Transmission Line Impedance (2)

Power on a lossless line is computed from the voltage and current

$$V(z) = V_o^+ e^{-jbz} + \Gamma V_o^+ e^{+jbz} = V_{\text{inc}} + V_{\text{ref}}$$

$$I(z) = \underbrace{\frac{V_o^+}{Z_o} e^{-jbz}}_{\substack{\text{INCIDENT} \\ \text{WAVE}}} - \underbrace{\Gamma \frac{V_o^+}{Z_o} e^{+jbz}}_{\substack{\text{REFLECTED} \\ \text{WAVE}}} = I_{\text{inc}} + I_{\text{ref}}$$

The incident instantaneous power in the incident wave is

$$P_i = \Re\{V_i\, e^{jwt}\}\Re\{I_i\, e^{jwt}\}$$

$$= \Re\left\{|V_o^+|\, e^{j\Phi_o} e^{jwt}\right\}\Re\left\{\frac{|V_o^+|\, e^{j\Phi_o}}{Z_o} e^{jwt}\right\} = \frac{|V_o^+|^2}{Z_o}\cos^2(wt + \Phi_o)$$

where it has been assumed that Z_o is real. A similar analysis of the reflected wave yields

$$P_r = -\frac{|V_o^+|^2}{Z_o}|\Gamma|\cos^2(wt + \Phi_o + \Phi_\Gamma)$$

Transmission Line Impedance (3)

The time-averaged power is obtained by integrating the instantaneous value over a period

$$P_{\mathrm{av}_i} = \frac{1}{T}\int_0^T P_i(t)dt = \frac{|V_o^+|^2}{Z_o(1/f)}\int_0^{1/f}\cos^2\left(wt+\Phi_o\right)dt = \frac{w}{2p}\frac{p}{w}\frac{|V_o^+|^2}{Z_o} = \frac{|V_o^+|^2}{2Z_o}$$

Similarly for the reflected power

$$P_{\mathrm{av}_r} = -\left|\Gamma\right|^2 P_{\mathrm{av}_i}$$

and the average power delivered to the load is

$$P_{\mathrm{av}_L} = P_{\mathrm{av}_i} + P_{\mathrm{av}_r} = \frac{|V_o^+|^2}{Z_o}\left(1-\left|\Gamma\right|^2\right)$$

In order to deliver all power to the load we must have $\left|\Gamma\right|^2 \to 0$.

Transmission Line Impedance (4)

Input impedances for the special load conditions

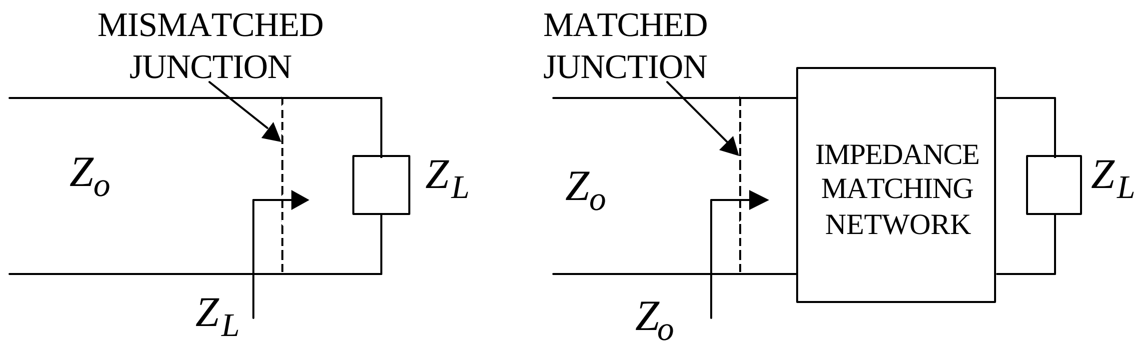
1. Short circuit: $Z_{\text{in}}^{\text{sc}} = Z_o \left[\frac{0 + jZ_o \tan(b\ell)}{Z_o + j0 \tan(b\ell)} \right] = jZ_o \tan(b\ell) \underset{b\ell \to 0}{\approx} jZ_o b\ell$ which is <u>inductive</u>
2. Open circuit: $Z_{\text{in}}^{\text{oc}} = Z_o \left[\frac{\infty + jZ_o \tan(b\ell)}{Z_o + j\infty \tan(b\ell)} \right] = \frac{Z_o}{j \tan(b\ell)} \underset{b\ell \to 0}{\approx} - jZ_o / b\ell$ which is <u>capacitive</u>
3. Matched line: $Z_{\text{in}} = Z_o \left[\frac{Z_o + jZ_o \tan(b\ell)}{Z_o + jZ_o \tan(b\ell)} \right] = Z_o$ (Note it is independent of the line length.)

Input impedances for some special line lengths:

1. Half-wavelength line: $Z_{\text{in}} = Z_o \left[\frac{Z_L + jZ_o \tan(p)}{Z_o + jZ_L \tan(p)} \right] = Z_L$
2. Quarter-wavelength line: $Z_{\text{in}} = Z_o \left[\frac{Z_L + jZ_o \tan(p/2)}{Z_o + jZ_L \tan(p/2)} \right] = Z_o^2 / Z_L$

Impedance Matching

For "off-the-shelf" components that must be used in a system, fixed values of Z_o are used. Common values are 50, 75 and 300 ohms. Most devices (antennas, couplers, phase shifters, etc.) are not "naturally" 50 ohms. An impedance matching circuit must be inserted between the 50 line and the device. The impedance matching circuit is usually incorporated into the device and sold as a single package as illustrated below.

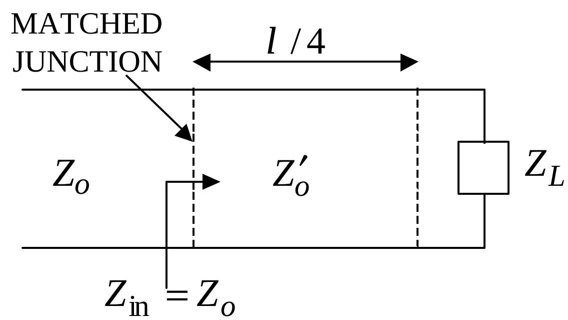


Three common matching techniques:

1. quarter-wave transformers
2. stub tuners
3. series and parallel lumped elements

In general, the imaginary component of the load impedance must be cancelled and the real part shifted to Z_o

Quarter-Wave Transformers (1)



If a quarter-wavelength section is inserted between the transmission line and load, the input impedance is

$$Z_{\text{in}} = Z_o' \left[\frac{Z_L + jZ_o' \tan(p/2)}{Z_o' + jZ_L \tan(p/2)} \right] = Z_o'^2 / Z_L \equiv Z_o \quad \Rightarrow \quad Z_o' = \sqrt{Z_o Z_L}$$

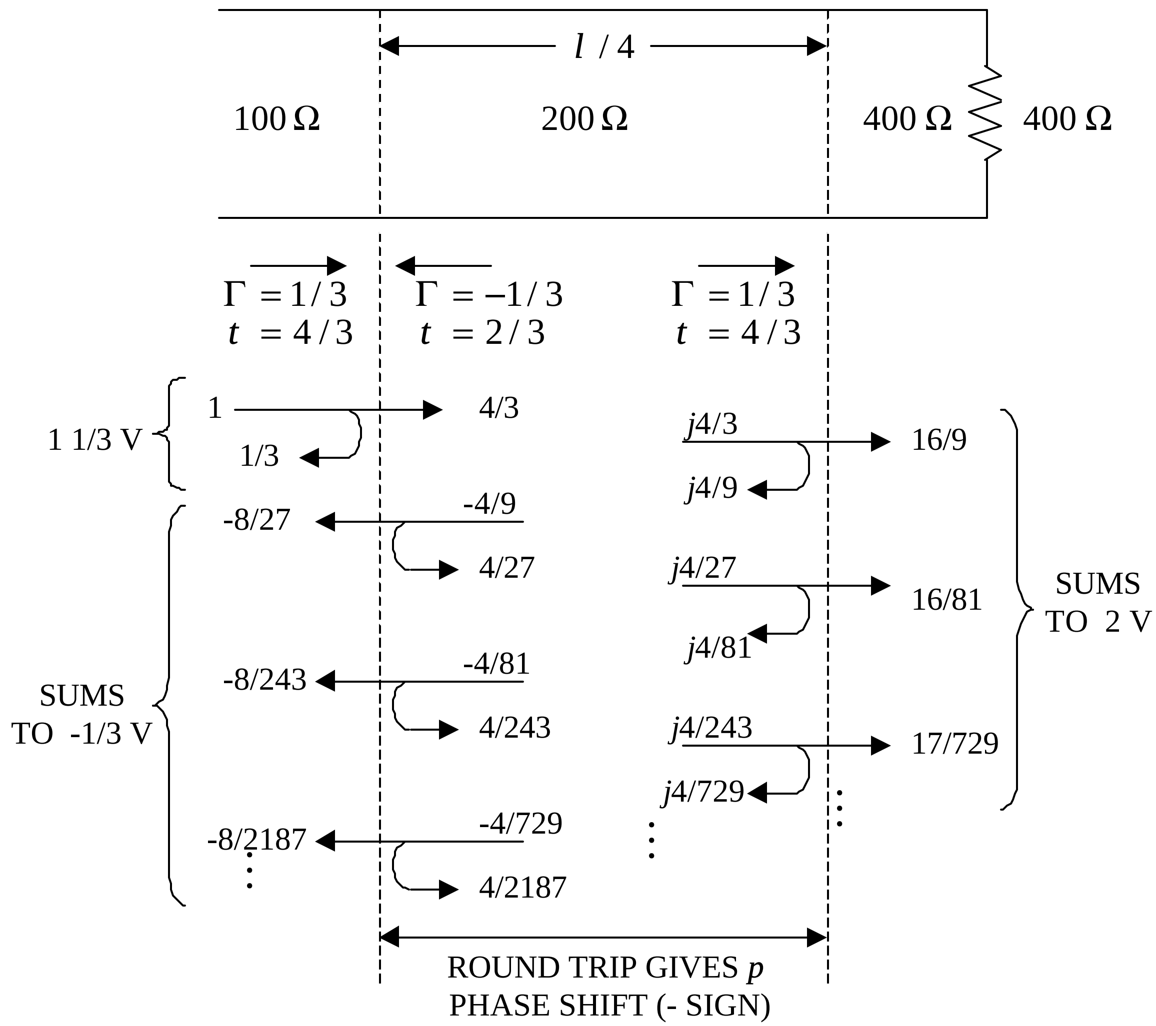
Note that all of the impedances involved must be real.

Example: What is the characteristic impedance of a quarter-wave section if it is to match a 100 ohm load to a 50 ohm line?

$$Z_o' = \sqrt{Z_o Z_L} = \sqrt{(50)(100)} = 70.7\ \Omega$$

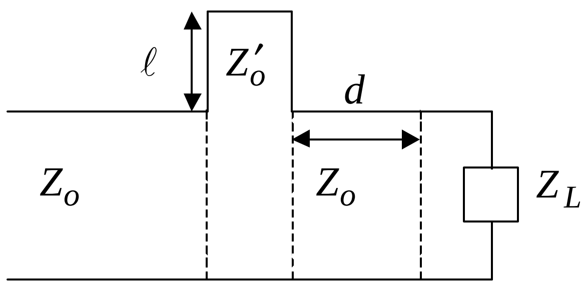
Quarter-Wave Transformers (2)

A quarter-wave transformer is designed so that reflections from the two junctions cancel (destructive interference). If the frequency is changed from its design value, then the cancellation is no longer complete.

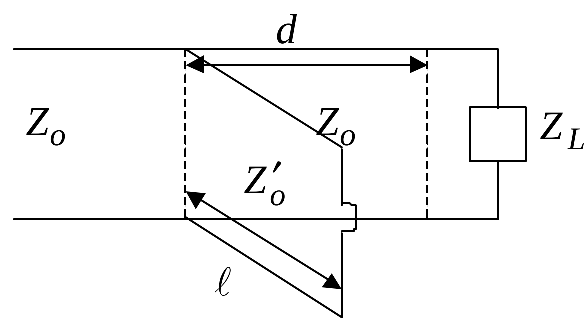


Stub Tuning (1)

We have seen that short sections of transmission line can be capacitive or inductive. By adjusting the location and length of a shorted or open stub, any complex impedance can be tuned.



SHORTED SERIES STUB



SHORTED PARALLEL STUB

- The stub impedance is usually chosen to be the same as that of the transmission line ($Z_o' = Z_o$).
- Both shorts and opens can be used. Shorts are generally preferred because there is less fringing of the field (and therefore less coupling to external objects)
- When dealing with devices in series we work with impedance, because the impedances of devices in series add: $Z = R + jX$ ($R =$resistance, $X =$reactance)
- When dealing with devices in parallel we work with admittance, because the admittances of devices in parallel add: $Y = G + jB$ ($G =$conductance, $B =$susceptance)

Stub Tuning (2)

Example of the parallel stub tuning process for a load with admittance

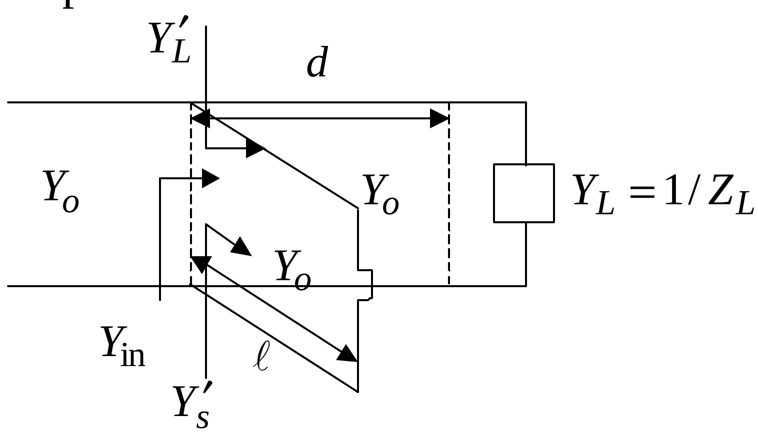
$$Y_L = G_L + jB_L = \frac{1}{Z_L} = \frac{1}{R_L + jX_L}$$

$Y_L' = G_L' + jB_L'$ = the admittance at distance d from the load, which can be found from the Z_{in} formula and then computing $Y_{\text{in}} = 1/Z_{\text{in}}$

$Y_s' = jB_s'$ = the admittance at a distance ℓ from the short (note that a shorted or open stub can provide only an imaginary part to the impedance or admittance)

The total admittance at the junction looking into the parallel combination of stub and load is $Y_{\text{in}} = Y_L' + Y_s' = G_L' + jB_L' + jB_s' \equiv Y_o$. Thus the two conditions are:

1. Choose d so that $G_L' = Y_o$
2. Choose ℓ so that $B_L' + B_s' = 0$

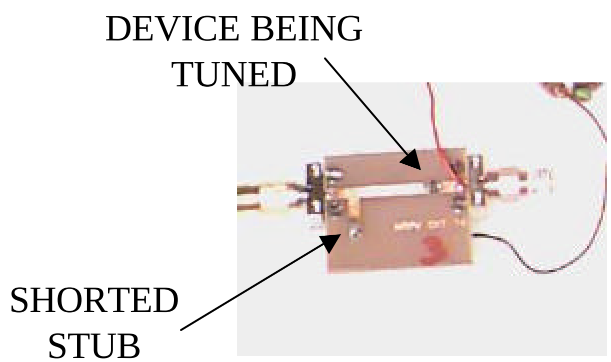


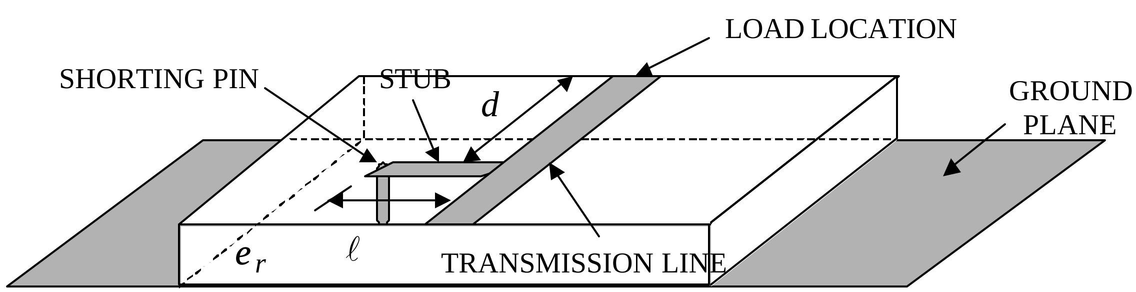
Stub Tuning (3)

The disadvantages of stub tuning:

- the required location and length of the stub may not be practical or convenient
- it is designed for a single frequency, and hence is narrow band

Example of short-circuited stub tuner on microstrip:

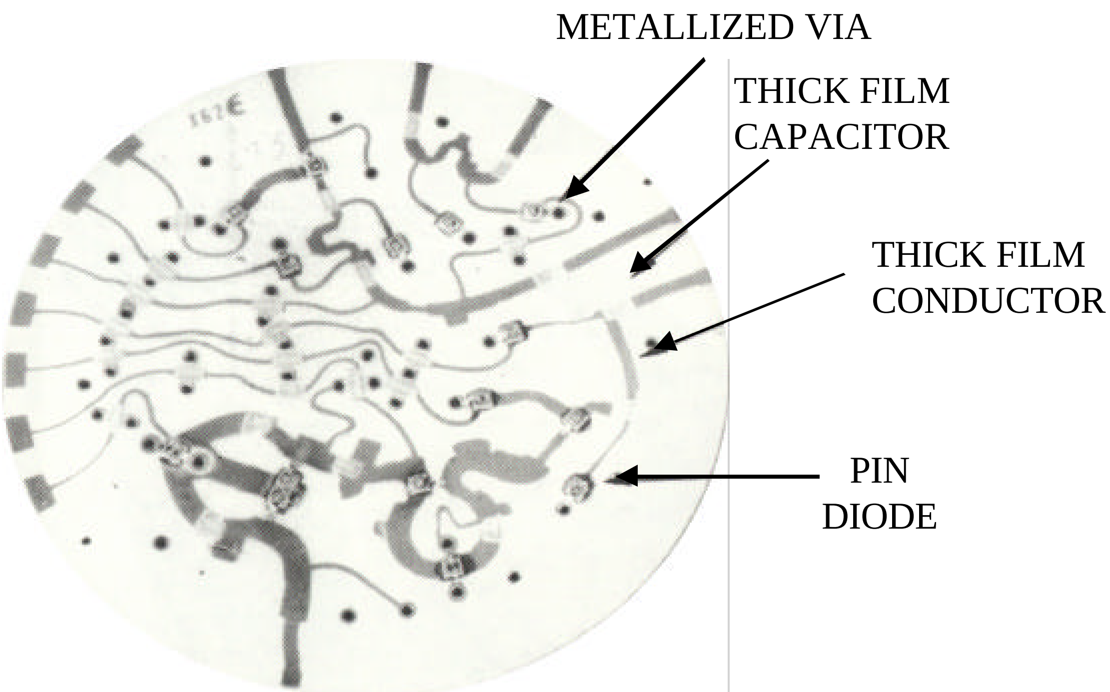




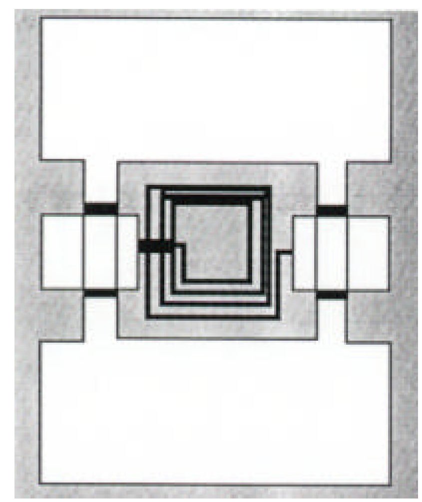
Lumped Element Tuning (1)

At low frequencies (about 1 GHz and below) lumped inductors, capacitors and resistors are effective and more compact that stubs.

Diode phase shifter with discrete components



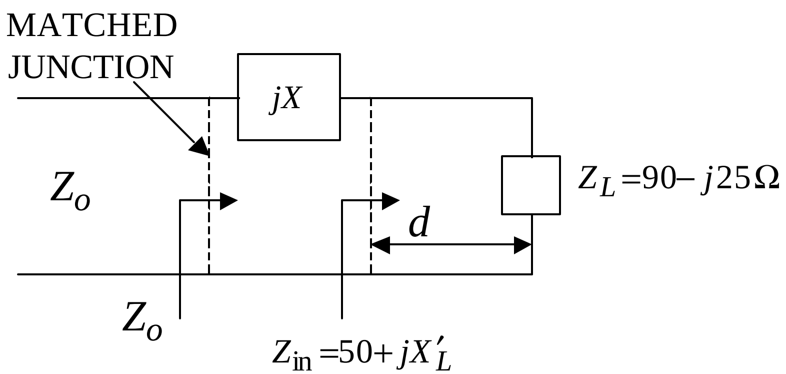
Printed circuit inductor



Lumped Element Tuning (2)

Example: An antenna with impedance $Z_L = 90 - j25$ ohms must be matched to a 50 ohm transmission line using an inductor or capacitor (no resistor). Design an impedance matching circuit.

The tuning element adds only a reactance, and therefore d must be chosen so that the impedance of the combination of load plus transmission line has a real part of 50.

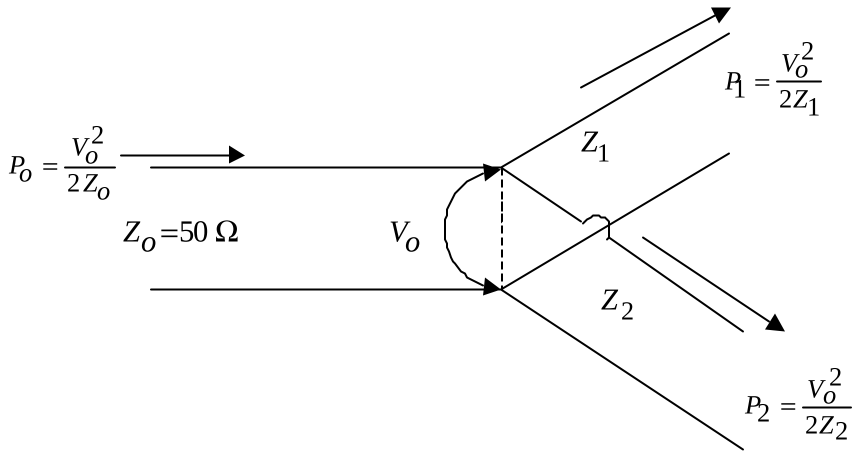


$$Z_{\text{in}} = Z_o \frac{Z_L + jZ_o \tan(bd)}{Z_o + jZ_L \tan(bd)}$$
$$= 50 + jX_L'$$

This equation can be separated into real and imaginary parts the real part solved for d. After much work one finds that the result is $d = 0.068\,l$. Using this in the above equation, the impedance is $Z_{\text{in}} = 50 - j35\,\Omega$. Therefore a series reactance of $+j35\,\Omega$ must be added, which is an inductor whose value is determined from $jwL = j35$.

Example: Lossless Power Divider

Design a lossless power divider that splits the power in the ratio of 2:1 between the two output arms.



Because the device is lossless $P_o = P_1 + P_2$. We want

$$P_1 = \frac{1}{3} P_o \quad \Rightarrow \quad \frac{V_o^2}{2Z_1} = \frac{1}{3}\frac{V_o^2}{2Z_o} \quad \Rightarrow \quad Z_1 = 3Z_o = 150\ \Omega$$

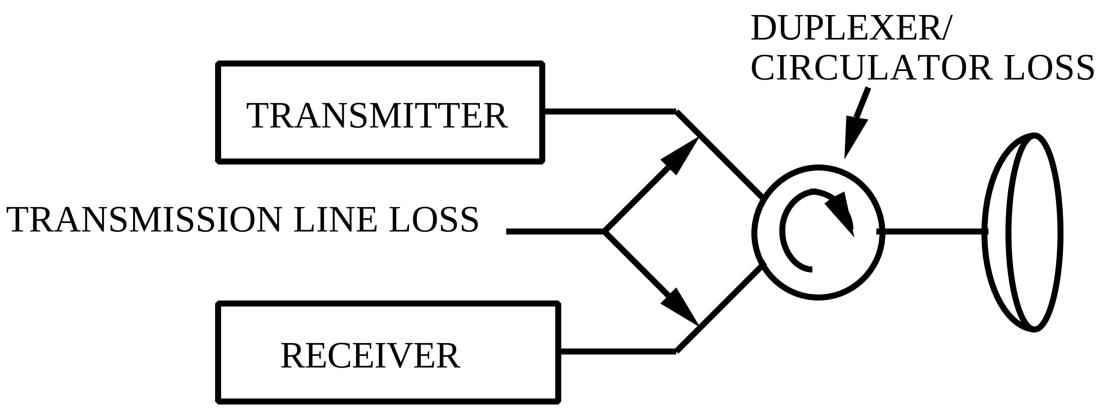
and, similarly,

$$P_2 = \frac{2}{3} P_o \quad \Rightarrow \quad \frac{V_o^2}{2Z_2} = \frac{2}{3}\frac{V_o^2}{2Z_o} \quad \Rightarrow \quad Z_2 = \frac{3}{2} Z_o = 75\ \Omega$$

Transmission Line Loss (1)

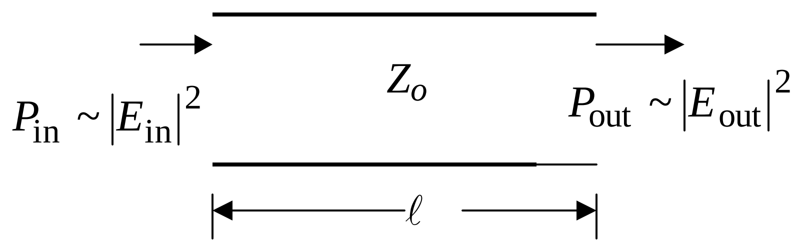
Transmission lines between the antenna and receiver or transmitter in a systems can have significant losses. Traditionally they have been called plumbing loss because the primary contributor was long sections of wavguide. Sources of loss include:

1. cables and waveguide runs (0.25 to 1 dB per meter)
2. devices have insertion loss
   duplexer, rotary joints, filters, switches, etc.
3. devices and connectors have mismatch loss (VSWR $\neq$ 1)



Transmission Line Loss (2)

Consider a length ℓ of transmission line



If the incident wave is TEM, then the field at the output can be expressed as $|E_{\text{out}}| = |E_{\text{in}}| e^{-a\ell}$ and the transmission coefficient of the section is

$$|t| = \frac{|E_{\text{out}}|}{|E_{\text{in}}|} = e^{-a\ell} \quad \Rightarrow \quad t_{\text{dB}} = 20\log\left(\left|\frac{E_{\text{out}}}{E_{\text{in}}}\right|\right) = 20\log\left(e^{-a\ell}\right)$$

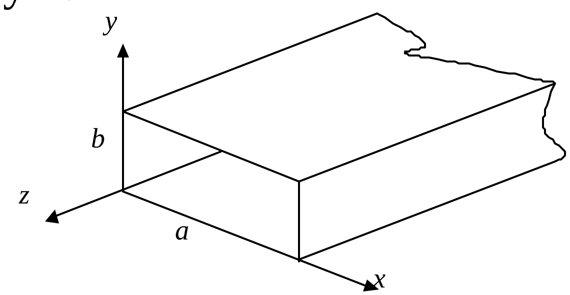
Example: A shorted 5m section of transmission line has 8 dB of loss. What is the attenuation coefficient?

Because the line is shorted the wave travels 10m and therefore,

$$20\log\left(e^{-10a}\right) = -8 \quad \Rightarrow \quad e^{-10a} = 10^{-(8/20)} \quad \Rightarrow \quad a = \frac{\ln\left(10^{-0.4}\right)}{-10} = 0.092 \text{ Np/m}$$

Waveguides (1)

Waveguides are an efficient means of transmitting microwaves. They can be hollow or filled with dielectric or other material. The cross section can be of any shape, but rectangular and circular are most common. First, we examine propagation in a rectangular waveguide of dimension *a* by *b*.



Waves propagate in the $\pm z$ direction: $\vec{E}(z),\ \vec{H}(z) \sim e^{\pm jbz}$. First separate Maxwell's equations into cartesian components (m, e refer to the material inside of the waveguide)

$$\left.\begin{array}{r}
\dfrac{\partial E_z}{\partial y} + jbE_y = -jwmH_x \\[2ex]
-jbE_x - \dfrac{\partial E_z}{\partial x} = -jwmH_y \\[2ex]
\dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -jwmH_z
\end{array}\right\} \nabla \times \vec{E} = -jwm\vec{H}$$

Waveguides (2)

$$\left.\begin{aligned}
&\frac{\partial H_z}{\partial y} + jbH_y = jweE_x \\
&-jbH_x - \frac{\partial H_z}{\partial x} = jweE_y \\
&\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = jweE_z
\end{aligned}\right\} \nabla \times \vec{H} = jwe\vec{E}$$

Rearranging

$$E_x = \frac{-j}{w^2 me - b^2}\left(b\frac{\partial E_z}{\partial x} + wm\frac{\partial H_z}{\partial y}\right)$$

$$E_y = \frac{j}{w^2 me - b^2}\left(-b\frac{\partial E_z}{\partial y} + wm\frac{\partial H_z}{\partial x}\right)$$

$$H_x = \frac{j}{w^2 me - b^2}\left(we\frac{\partial E_z}{\partial y} - b\frac{\partial H_z}{\partial x}\right)$$

$$H_y = \frac{-j}{w^2 me - b^2}\left(we\frac{\partial E_z}{\partial x} + b\frac{\partial H_z}{\partial y}\right)$$

Waveguides (3)

The wave equations are:

$$\nabla^2 \vec{E} = -w^2 me \vec{E}$$
$$\nabla^2 \vec{H} = -w^2 me \vec{H}$$

Note that $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and $\frac{\partial^2}{\partial z^2} = (-jb)^2 = -b^2$ and the wave equations for the *z* components of the fields are

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) E_z = \left(b^2 - w^2 me\right) E_z$$
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) H_z = \left(b^2 - w^2 me\right) H_z$$

<u>TEM waves do not exist in hollow rectangular waveguides</u>. The wave equations must be solved subject to the boundary conditions at the waveguide walls. We consider two types of solutions for the wave equations: (1) transverse electric (TE) and (2) transverse magnetic (TM).

Waveguides (4)

<u>Transverse magnetic</u> (TM) waves: $H_z = 0$ and thus $\vec{H}$ is transverse to the z axis. All field components can be determined from E_z. The general solution to the wave equation is

$$\begin{aligned} E_z(x,y,z) &= E_z(x,y)e^{\pm jbz} = E_z(x)E_z(y)e^{\pm jbz} \\ &= \left(A\cos(b_x x) + B\sin(b_x x)\right)\left(C\cos\left(b_y y\right) + D\sin\left(b_y y\right)\right)e^{\pm jbz} \end{aligned}$$

where A, B, C, and D are constants. The boundary conditions must be satisfied:

$$E_z = 0 \text{ at } \begin{cases} x = 0 \rightarrow A = 0 \\ y = 0 \rightarrow C = 0 \end{cases}$$

Choose b_x and b_y to satisfy the remaining conditions.

$$E_z = 0 \text{ at } x = a: \quad \sin\left(b_x a\right) = 0 \quad \Rightarrow \quad b_x a = mp \quad \Rightarrow \quad b_x = \frac{mp}{a} \quad (m = 1,2,\ldots)$$

$$E_z = 0 \text{ at } y = b: \quad \sin\left(b_x b\right) = 0 \quad \Rightarrow \quad b_y b = np \quad \Rightarrow \quad b_y = \frac{np}{b} \quad (n = 1,2,\ldots)$$

Waveguides (5)

For TM waves the longitudinal component of the electric field for a +*z* traveling wave is given by

$$E_z(x, y, z) = U \sin\left(\frac{mp}{a} x\right) \sin\left(\frac{np}{b} y\right) e^{-jbz}$$

where the product of the constants *AB* has been replaced by a new constant U. Each solution (i.e., combination of *m* and *n*) is called a <u>mode</u>. Now insert E_z back in the wave equation to obtain a <u>separation equation</u>:

$$b^2 = w^2 me - \left(\frac{mp}{a}\right)^2 - \left(\frac{np}{b}\right)^2$$

If $b^2 > 0$ then propagation occurs; $b^2 = 0$ defines a <u>cuttoff frequency</u>, $f_{c_{mn}}$,

$$f_{c_{mn}} = \frac{1}{2\sqrt{me}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Waves whose frequencies are above the cutoff frequency for a mode will propagate, but those below the cutoff frequency are attenuated.

Waveguides (6)

<u>Transverse electric</u> (TE) waves: $E_z = 0$ and thus $\vec{E}$ is transverse to the z axis. All field components can be determined from H_z. The general solution to the wave equation is

$$H_z(x,y,z) = H_z(x,y)e^{\pm jbz} = H_z(x)H_z(y)e^{\pm jbz}$$
$$= \left(A\cos(b_x x) + B\sin(b_x x)\right)\left(C\cos(b_y y) + D\sin(b_y y)\right)e^{\pm jbz}$$

But, from Maxwell's equations, $E_x \propto \dfrac{\partial H_z}{\partial y} \sim \cos\left(\dfrac{np}{b} y\right)$ and $E_y \propto \dfrac{\partial H_z}{\partial x} \sim \cos\left(\dfrac{mp}{a} x\right)$.

Boundary conditions:

$$E_x = 0 \text{ at } y = 0 \rightarrow D = 0$$
$$E_y = 0 \text{ at } x = 0 \rightarrow B = 0$$
$$E_x = 0 \text{ at } y = b \rightarrow b_y = \frac{np}{b}, \quad n = 0,1,\ldots$$
$$E_y = 0 \text{ at } x = a \rightarrow b_x = \frac{mp}{a}, \quad m = 0,1,\ldots$$

Therefore, $$H_z(x,y,z) = V\cos\left(\frac{mp}{a}x\right)\cos\left(\frac{np}{b}y\right)e^{-jbz} \quad (m = n = 0 \text{ not allowed})$$

The same equation for cutoff frequency holds for both TE and TM waves.

Waveguides (7)

Other important relationships:

- Phase velocity for mode (*m*,*n*), $u_p = \dfrac{u}{\sqrt{1-\left(f_{c_{mn}} / f\right)^2}}$ where $u = 1/\sqrt{me}$ is the phase velocity in an unbounded medium of the material which fills the waveguide. Note the the phase velocity in the waveguide is larger than in the unbounded medium (and can be greater than *c*).
- Group velocity for mode (*m*,*n*), $u_g = u\sqrt{1-\left(f_{c_{mn}} / f\right)^2}$. This is the velocity of energy (information) transport and is less than the velocity in the unbounded medium.
- Wave impedance for mode (*m*,*n*),

$$Z_{\mathrm{TE}_{mn}} = \frac{h}{\sqrt{1-\left(f_{c_{mn}} / f\right)^2}}$$

$$Z_{\mathrm{TM}_{mn}} = h\sqrt{1-\left(f_{c_{mn}} / f\right)^2}$$

where $h = \sqrt{m/e}$ is the wave impedance in the unbounded medium.

- Phase constant for mode (*m*,*n*), $b_{mn} = \dfrac{w}{u_p} = \dfrac{w}{u}\sqrt{1-\left(f_{c_{mn}} / f\right)^2}$

Waveguides (8)

- Guide wavelength for mode (m,n), $l_{g_{mn}} = \dfrac{l}{\sqrt{1-\left(f_{c_{mn}} / f\right)^2}}$ where l is the wavelength in the unbounded medium.

The dominant mode is the one with the lowest cutoff frequency. For rectangular waveguides with $a > b$ the TE_{10} mode is dominant. If a mode shares a cutoff frequency with another mode(s), then it is degenerate. For example, TE_{11} and TM_{11} are degenerate modes.

Example: If the following field exists in a rectangular waveguide what mode is propagating?

$$E_z = 5\sin\left(\frac{2p}{a}x\right)\sin\left(\frac{p}{b}y\right)e^{-j2z}$$

Since $E_z \neq 0$ it must be a TM mode. Compare it with the general form of a TM mode field and deduce that m=2 and n=1. Therefore, it is the TM_{21} mode.

Waveguides (9)

Example: What is the lowest frequency that will readily propagate through a tunnel with a rectangular cross section of dimension 10m by 5m?

If the walls are good conductors, we can consider the tunnel to be a waveguide. The lowest frequency will be that of the dominant mode, which is the TE_{10} mode. Assume that the tunnel is filled with air

$$f_{c_{10}} = \frac{1}{2\sqrt{\mu_o \epsilon_o}}\left(\frac{1}{a}\right) = \frac{c}{2(10)} = 15 \text{ MHz}$$

Example: Find the five lowest cutoff frequencies for an air-filled waveguide with a=2.29 cm and b=1.02 cm.

$$f_{c_{mn}} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{0.029}\right)^2 + \left(\frac{n}{0.0102}\right)^2}$$

Use Matlab to generate cutoff frequencies by looping through m and n. Choose the five lowest. Note that when both m,n > 1 then both TE and TM modes must be listed. (The frequencies are listed in GHz.)

$$\mathrm{TE}_{01}(14.71), \mathrm{TE}_{10}(6.55), \mathrm{TE}_{11} \text{ and } \mathrm{TM}_{11}(16.10), \mathrm{TE}_{20}(13.10)$$

Waveguides (10)

Example: Find the field parameters for a TE_{10} mode, f=10 GHz, a=1.5 cm, b=0.6 cm, filled with dielectric, $e_r = 2.25$.

Phase velocity in the unbounded medium, $u = c/\sqrt{2.25} = 3\times10^8/1.5 = 2\times10^8$ m/s

Wavelength in the unbounded medium, $l = u/f = 2\times10^8/1\times10^{10} = 0.02$ m

Cutoff frequency, $f_{c_{10}} = u/(2a) = \dfrac{c/\sqrt{2.25}}{(2)(0.015)} = 0.67\times10^{10}$ Hz

Phase constant, $b_{10} = \dfrac{w}{u}\sqrt{1-\left(f_{c_{mn}}/f\right)^2} = \dfrac{2pf}{c/\sqrt{2.25}}\underbrace{\sqrt{1-(0.067/1)^2}}_{0.745} = 74.5p$ radians

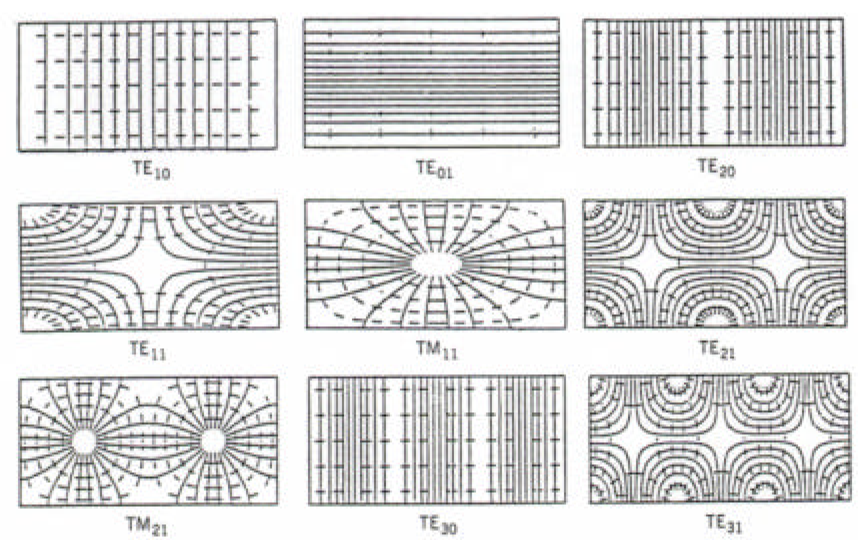
Guide wavelength, $l_g = \dfrac{l}{\sqrt{1-\left(f_{c_{mn}}/f\right)^2}} = \dfrac{0.02}{0.745} = 0.0268$ m

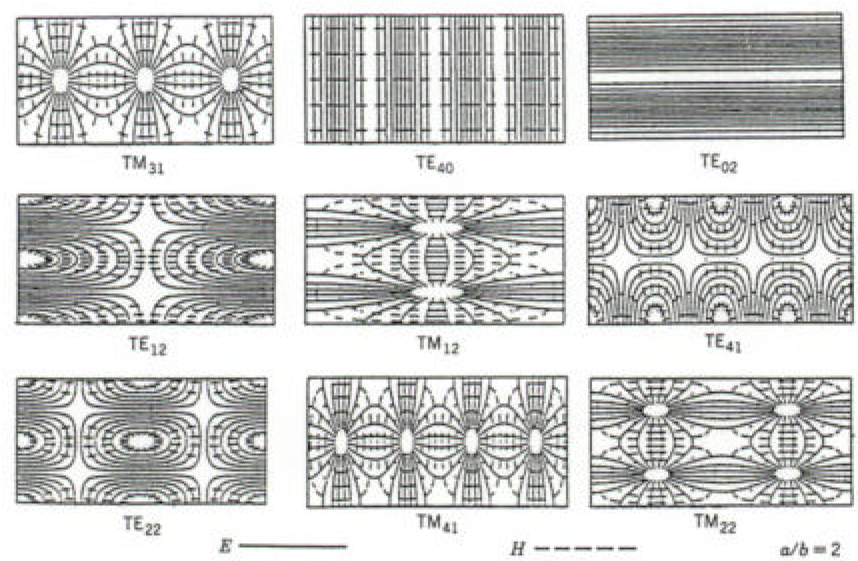
Phase velocity, $u_p = u/0.745 = 2\times10^8/0.745 = 2.68\times10^8$ m/s

Wave impedance, $Z_{\mathrm{TE}_{10}} = \dfrac{h}{\sqrt{1-\left(f_{c_{mn}}/f\right)^2}} = \dfrac{h_o/\sqrt{2.25}}{0.745} = \dfrac{(377)}{(0.745)(1.5)} = 337.4$ ohms

Group velocity, $u_g = 0.745u = (2\times10^8)(0.745) = 1.49\times10^8$ m/s

Mode Patterns in Rectangular Waveguide





From C.S. Lee, S. W. Lee, and L. L. Chuang, "Plot of Modal Field Distribution in Rectangular and Circular Waveguides," *IEEE Trans. on MTT*, 1985.

Table of Waveguide Formulas

QUANTITY	TEM ($E_Z = H_Z = 0$)	TM ($H_Z = 0$)	TE ($E_Z = 0$)
WAVE IMPEDANCE, Z	$Z_{\text{TEM}} = h = \sqrt{\frac{m}{e}}$	GENERAL: $Z_{\text{TM}} = \frac{g}{jwe}$ $f > f_c$: $h\sqrt{1-(f_c/f)^2}$ $f < f_c$: $\frac{-jh}{we}\sqrt{1-(f/f_c)^2}$	GENERAL: $Z_{\text{TE}} = \frac{jwm}{g}$ $f > f_c$: $\frac{h}{\sqrt{1-(f_c/f)^2}}$ $f < f_c$: $\frac{jwm}{h\sqrt{1-(f/f_c)^2}}$
PROPAGATION CONSTANT, g	$jk = jw\sqrt{me}$	GENERAL: $h\sqrt{1-(f_c/f)^2}$ $f > f_c$: $jb = jk\sqrt{1-(f_c/f)^2}$ $f < f_c$: $a = h\sqrt{1-(f/f_c)^2}$	GENERAL: $h\sqrt{1-(f_c/f)^2}$ $f > f_c$: $jb = jk\sqrt{1-(f_c/f)^2}$ $f < f_c$: $a = h\sqrt{1-(f/f_c)^2}$
PHASE VELOCITY, u_p	$u = \frac{1}{\sqrt{me}}$	GENERAL: w/b $f > f_c$: $\frac{u}{\sqrt{1-(f_c/f)^2}}$ $f < f_c$: NO PROPAGATION	GENERAL: w/b $f > f_c$: $\frac{u}{\sqrt{1-(f_c/f)^2}}$ $f < f_c$: NO PROPAGATION
VECTOR FIELD RELATIONSHIP	$\vec{H} = \frac{1}{Z_{\text{TEM}}}\hat{k}\times\vec{E}$	$\vec{E}_T = -\frac{g}{h^2}\nabla_T E_z$ $\vec{H} = \frac{1}{Z_{\text{TM}}}\hat{z}\times\vec{E}$	$\vec{H}_T = -\frac{g}{h^2}\nabla_T H_z$ $\vec{E} = -Z_{\text{TE}}\hat{z}\times\vec{H}$

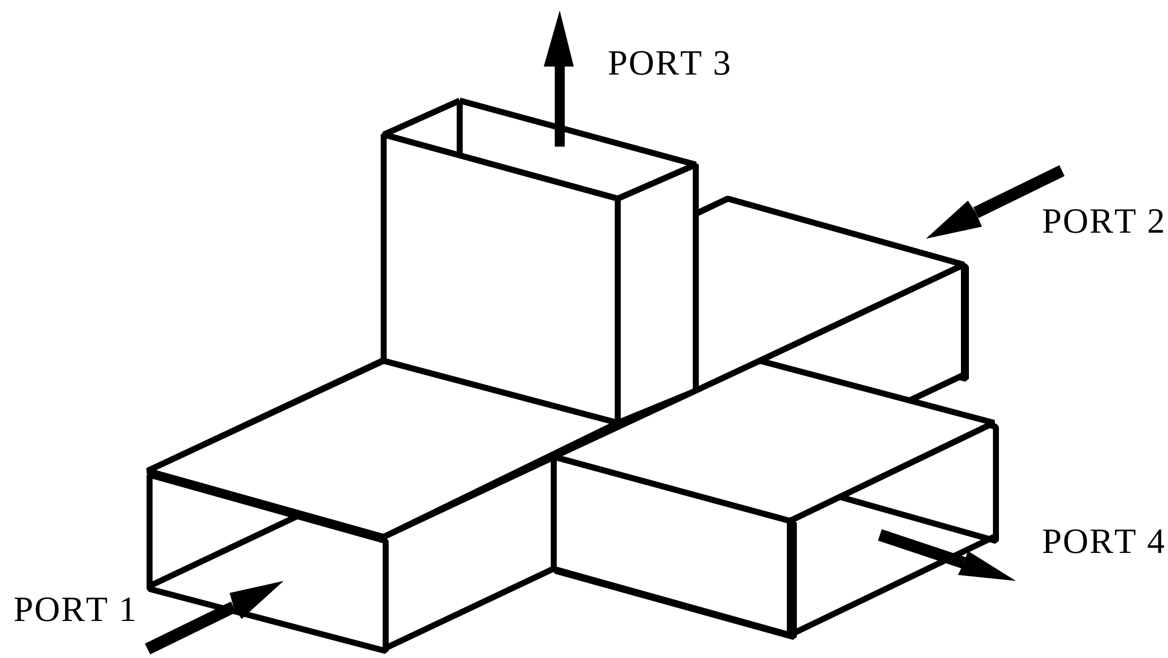
Cutoff frequency: $f_c = \frac{h}{2p\sqrt{me}}$ Propagation constant: $g = \sqrt{h^2 - k^2}$ Transverse Laplacian: $\nabla_T^2 = \frac{\P^2}{\P x^2} + \frac{\P^2}{\P y^2}$

For a rectangular waveguide (a by b): $h = \sqrt{\left(\frac{mp}{a}\right)^2 + \left(\frac{np}{b}\right)^2}$ Guide wavelength: $l_g = \frac{l}{\sqrt{1-(f_c/f)^2}}$

Waveguide Magic Tee

Ports 1 and 2 are the "sidearms." Port 4 is the "sum" port and 3 the "difference" port.

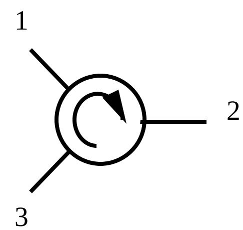
Sidearm excitation	Port 3	Port 4
$A_1 = ae^{jf},\ A_2 = ae^{jf}$	$A_3 = 0$	$A_4 = 2a$
$A_1 = ae^{jf},\ A_2 = ae^{jf+p}$	$A_3 = 2a$	$A_4 = 0$



"Magic" originates from the fact that it is the only 4-port device that can be simultaneously matched at all ports.

Circulators

Circulators "circulate" the signal from port to port in the direction indicated by the arrow



Ideally: Signal into port 1 emerges out port 2; signal out port 3 is zero.
Signal into port 2 emerges out port 3; signal out port 1 is zero.
Signal into port 3 emerges out port 1; signal out port 2 is zero.

In practice:

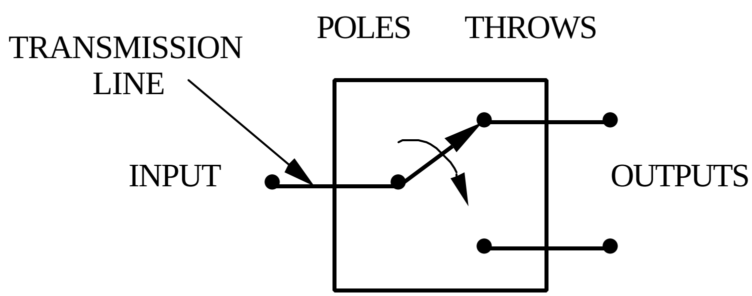
1. There is some insertion loss in the forward (arrow) direction. Values depend on the type of circulator. They range from 0.5 dB to several dB.
2. There is leakage in the reverse (opposite arrow) direction. Typical values of <u>isolation</u> are 20 to 60 dB. That is, the <u>leakage signal</u> is 20 to 60 dB below the signal in the forward direction.
3. Increasing the isolation comes at the expense of size and weight

Uses: 1. Allow a transmitter and receiver to share a common antenna without switching
2. Attenuate reflected signals (load the third port)

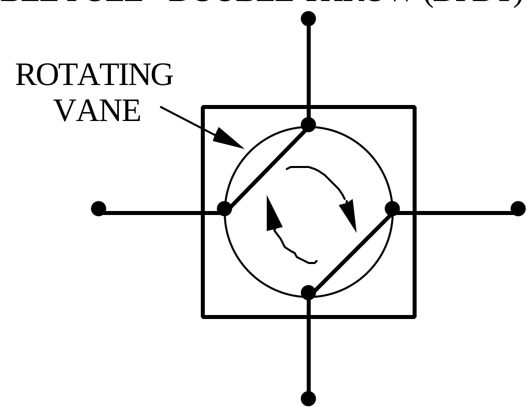
Microwave Switches

Microwave switches are used to control signal transmission between circuit devices. A general representation of a switch is given in terms of "poles" and "throws"

SINGLE POLE - DOUBLE THROW (SPDT)



DOUBLE POLE - DOUBLE THROW (DPDT)

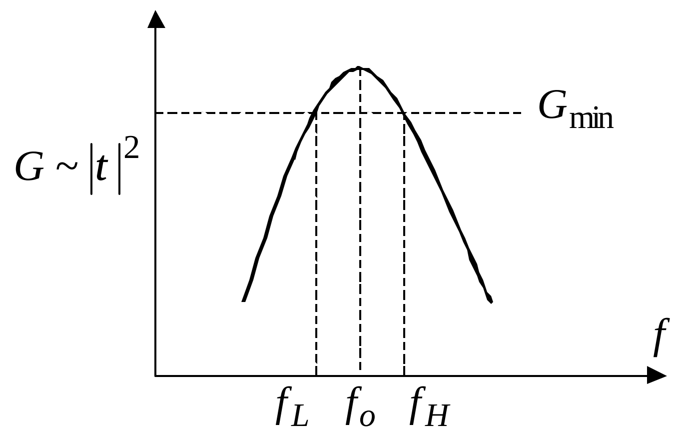


Switches can be constructed in any type of transmission line or waveguide. Common types:

Type	Principle	Applied to:
Mechanical	Rotating or moving parts	All types
Diode	Forward/backward bias yields low/high impedance	Stripline, microstrip, waveguide
Gas discharge	Confined gas is ionized	Waveguide
Circulator	Magnetized ferrite switches circulation direction	Stripline, microstrip, waveguide

Bandwidth (1)

The equivalent circuits of transmission lines and antennas are comprised of combinations of resistors, capacitors and inductors. The transmission coefficient, or gain in the case of an antenna, is frequency dependent. The range of frequencies over which the device has "acceptable performance" is called the bandwidth of the device. For example, the gain of a typical antenna has the following general frequency characteristic:



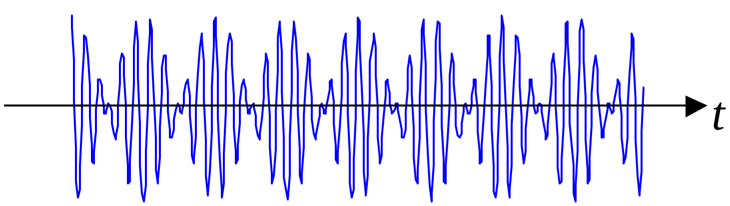
Note that gain can be viewed as a scaled value of the antenna's transmission coefficient. We will see that other performance measures, not just gain will determine its bandwidth. Specifying frequencies where the gain exceeds the minimum value as in the operating band, the bandwidth is $f_H - f_L$. The center of the band is $f_o = (f_H + f_L)/2$.

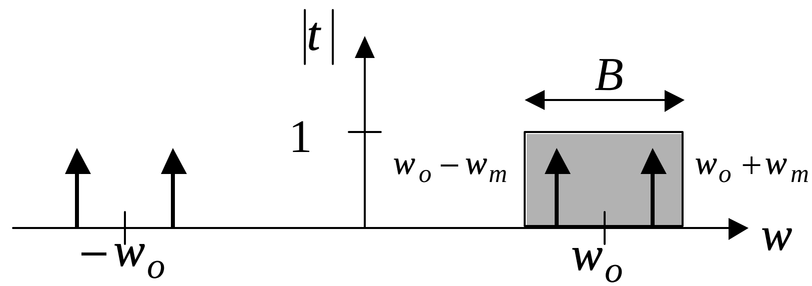
Bandwidth (2)

Information transmission systems, such as radar and communications, require a finite (non-zero) bandwidth. Consider the following waveform as an approximation to a modulated carrier that a radar would employ. In the time domain the signal is

$$s(t) = \cos(w_o t)\cos(w_m t) = \left(\frac{e^{jw_o t} + e^{-jw_o t}}{2}\right)\left(\frac{e^{jw_m t} + e^{-jw_m t}}{2}\right)$$

The spectrum of this signal has two spikes centered about the carrier frequency, $\pm w_o$

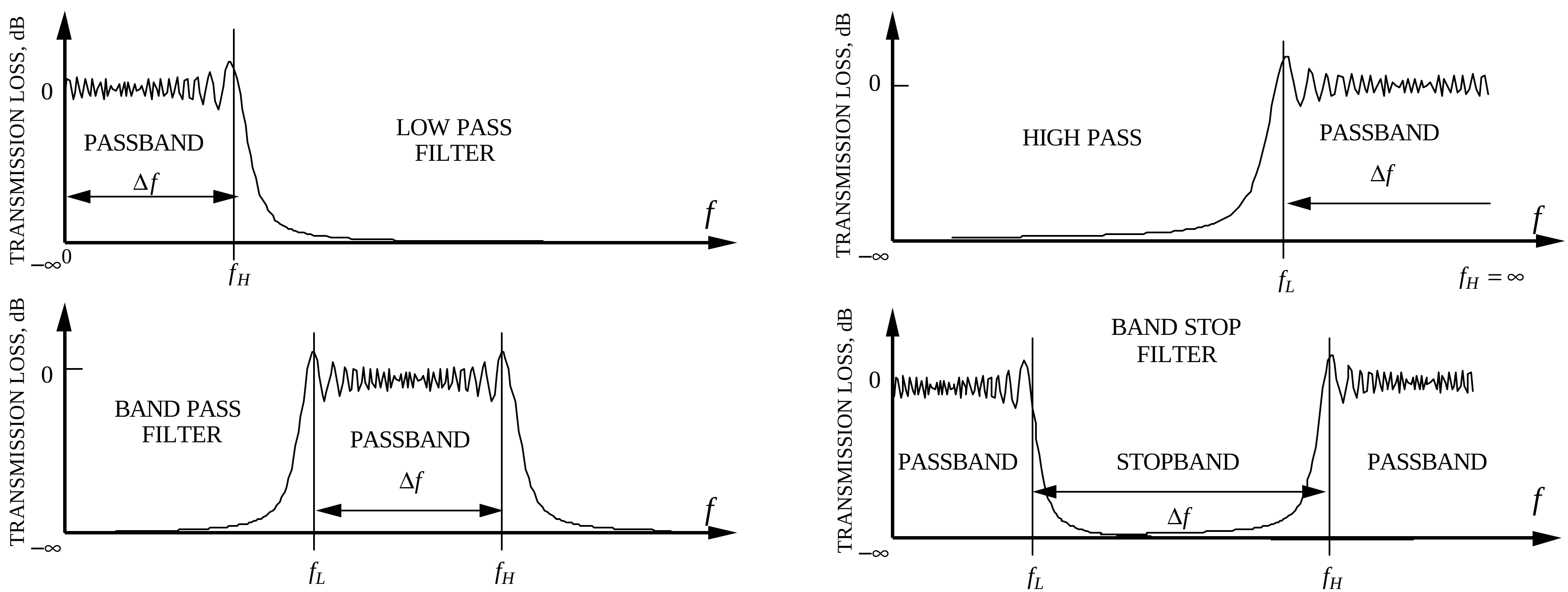




Therefore, in order to pass this signal without removing any frequency components, the required bandwidth is $B = \Delta f = 2w_m$. This is an example of a bandpass device. Ideally we would like the amplitude of the transmission coefficient to be constant over the passband. It is usually "bell-shaped" as depicted in the previous chart. Common cutoff choices for the edges of the band are the –3dB, –6dB, and –10dB points.

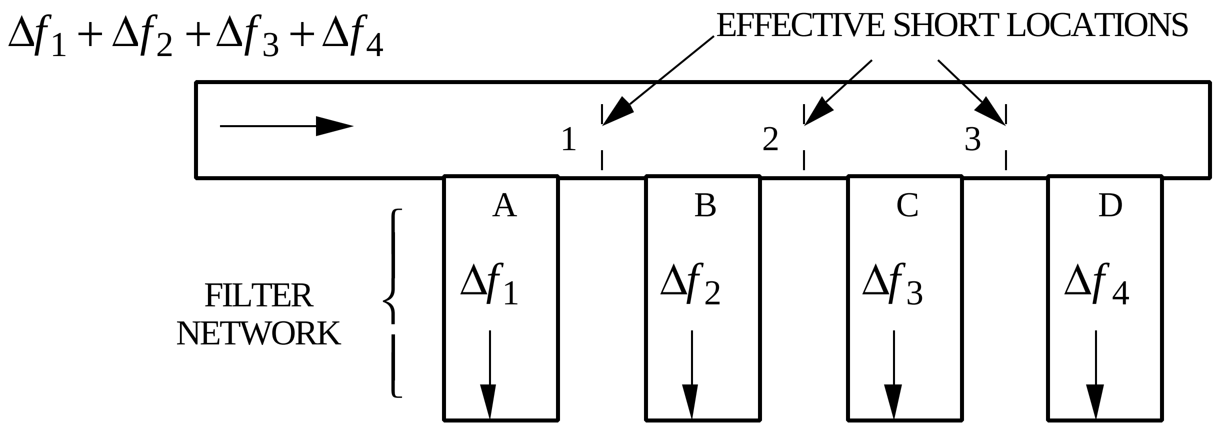
Filter Characteristics

Filters are characterized by their transfer functions $|H(f)| = |t| = \sqrt{1-|\Gamma|^2}$, where Γ is reflection coefficient. It is usually plotted as return loss in dB, $20\log_{10}(|\Gamma|)$, or transmission loss in dB, $20\log_{10}(|t|)$. Note that in many cases the phase of the characteristic function is also important.

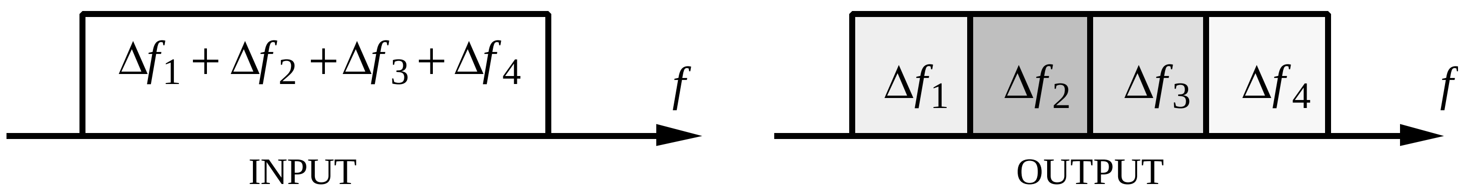


Multiplexers

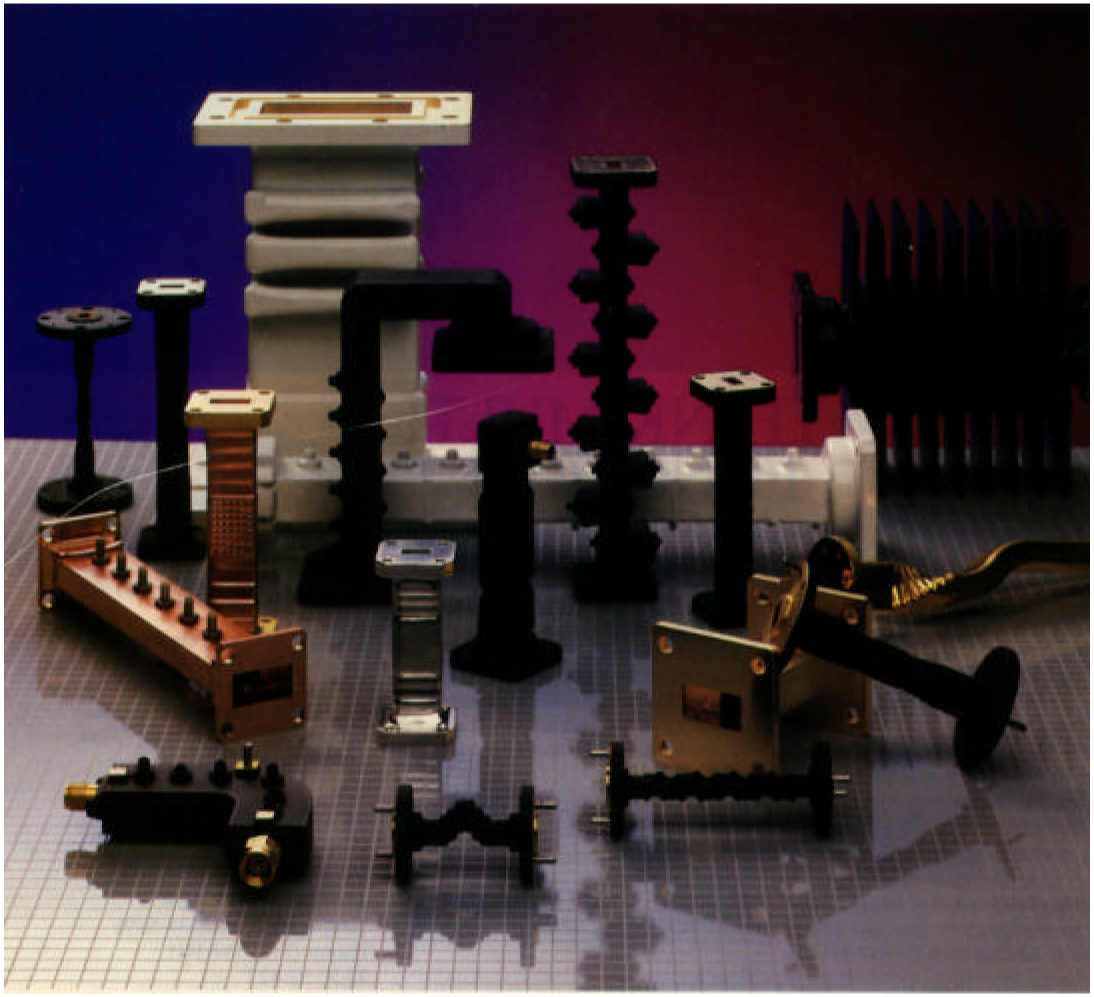
Multiplexers are frequency selective circuits used to separate signals by frequency spectrum. They are comprised of filter networks. The example illustrated is a waveguide manifold multiplexer that separates signals into four subbands:



The plane at 1 appears as a short in the band Δf_1, but matched at other frequencies . The waveguide junction at A appears matched at Δf_1, but shorted at other frequencies. Similarly for planes 2, 3, 4 and junctions B, C, D. Frequency characteristic:



Waveguide Filters



From *Gamma-F Corporation* advertisement

Decibel Refresher

In general, a dimensionless quantity Q in decibels (denoted Q_{dB}) is defined by

$$Q_{\text{dB}} = 10\log_{10}(Q)$$

Q usually represents a ratio of quantities, where the denominator is the reference. Characters are added to the "dB" to denote the reference quantity. For example:

- Power referenced to 1 watt: $\dfrac{P}{1\text{w}} = P_{\text{dBw}}$
- Power referenced to 1 milliwatt: $\dfrac{P}{1\text{mw}} = P_{\text{dBm}} (= P_{\text{dBw}} + 30)$
- Antenna gain referenced to an isotropic source: $\dfrac{G}{1} = G_{\text{dBi}}$

Recall that the gain of an ideal isotropic source is 1. This notation is not usually used because the definition of gain implies an isotropic reference. However, a dipole's gain (= 1.5) is sometimes is used as a reference, and the notation is dBd.

Note:
1. 10 dB represents an order magnitude change in the quantity Q
2. the dB unit does not depend on the reference that is used to define it
3. when quantities are multiplied their dB values add:

$$\text{ERP}_{\text{dBw}} = (PG)_{\text{dBw}} = P_{\text{dBw}} + G_{\text{dB}}$$

Coordinate Transform Tables

	$\hat{x}$	$\hat{y}$	$\hat{z}$
$\hat{r}$	$\cos f$	$\sin f$	0
$\hat{f}$	$-\sin f$	$\cos f$	0
$\hat{z}$	0	0	1

Rectangular and cylindrical

	$\hat{x}$	$\hat{y}$	$\hat{z}$
$\hat{r}$	$\sin q \cos f$	$\sin q \sin f$	$\cos q$
$\hat{q}$	$\cos q \cos f$	$\cos q \sin f$	$-\sin q$
$\hat{f}$	$-\sin f$	$\cos f$	0

Rectangular and spherical

	$\hat{r}$	$\hat{f}$	$\hat{z}$
$\hat{r}$	$\sin q$	0	$\cos q$
$\hat{q}$	$\cos q$	0	$-\sin q$
$\hat{f}$	0	1	0

Cylindrical and spherical

Example: from top table, reading across,

$$\hat{r} = \hat{x}\cos f + \hat{y}\sin f$$

and reading down,

$$\hat{x} = \hat{r}\cos f - \hat{f}\sin f$$

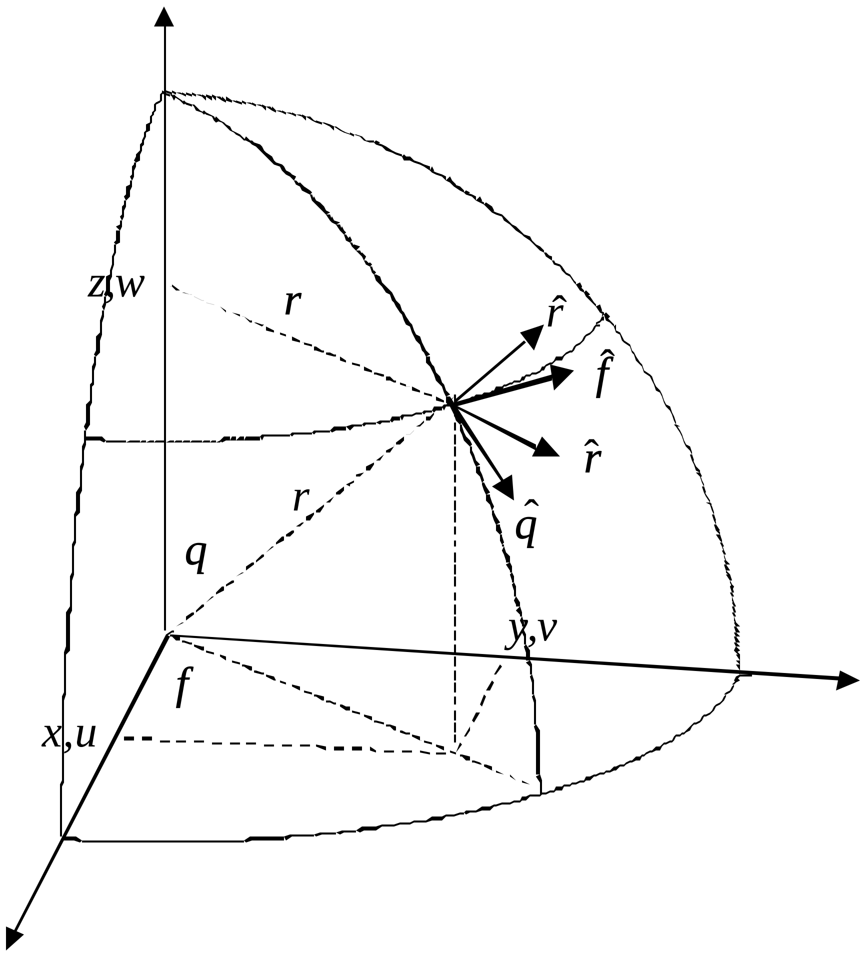
The tables also can be used to transform vectors. The unit vectors in the table headings are replaced by the corresponding vector components. For example, given

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

in cartesian coordinates, the vector can be expressed in cylindrical coordinates as

$$A_r = A_x \cos f + A_y \sin f + A_z \cdot 0$$

Coordinate Systems



$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$$

$$f = \tan^{-1}\left(\frac{y}{x}\right)$$

$$q = \tan^{-1}\left(\frac{r}{z}\right)$$

$$\vec{ds}_r = \hat{r} \sin q \, dq \, df$$

<u>Direction cosines</u> are the projections of points on the unit sphere onto the *xy* plane. They are the *x*,*y*, and *z* components of $\hat{r}$:

$$u = \sin q \cos f$$
$$v = \sin q \sin f$$
$$w = \cos q$$

Radar and ECM Frequency Bands

Standard Radar Bands[1]		ECM Bands[2]	
Band Designation[3]	**Frequency Range (MHz)**	**Band Designation**	**Frequency Range (MHz)**
HF	3–30	Alpha	0–250
VHF[4]	30–300	Bravo	250–500
UHF[4]	300–1,000	Charlie	500–1,000
L	1,000–2,000	Delta	1,000–2,000
S	2,000–4,000	Echo	2,000–3,000
C	4,000–8,000	Foxtrot	3,000–4,000
X	8,000–12,000	Golf	4,000–6,000
K_u	12,000–18,000	Hotel	6,000–8,000
K	18,000–27,000	India	8,000–10,000
K_a	27,000–40,000	Juliett	10,000–20,000
millimeter[5]	40,000–300,000	Kilo	20,000–40,000
		Lima	40,000–60,000
		Mike	60,000–100,000

[1] From IEEE Standard 521-1976, November 30 1976.
[2] From AFR 55-44 (AR105-96, OPNAVINST 3420.9B, MCO 3430.1), October 27, 1964.
[3] British usage in the past has corresponded generally but not exactly to the letter-designated bands.
[4] The following *approximate* lower frequency ranges are sometimes given letter designations: P-band (225–390 MHz), G-band (150–225 MHz), and I-band (100–150 MHz).
[5] The following *approximate* higher frequency ranges are sometimes given letter designations: Q-band (36–46 GHz), V-band (46–56 GHz), and W-band (56–100 GHz).

Electromagnetic Spectrum